

How to Value Bonds and Stocks

When the stock market closed on January 20, 2006, the common stock of McGraw-Hill, publisher of high-quality college textbooks, was going for \$49.34 per share. On that same day, Eastman Chemical closed at \$49.96, while pharmaceutical benefits manager Caremark Rx closed at \$50.02. Because the stock prices of these three companies were so similar, you might expect that they would be offering similar dividends to their stockholders, but you would be wrong. In fact, Eastman Chemical's dividend was \$1.76 per share,

McGraw-Hill's was \$0.66 per share, and Caremark Rx paid no dividends at all!

As we will see in this chapter, the dividends currently being paid are one of the primary factors we look at when attempting to value common stocks. However, it is obvious from looking at Caremark that current dividends are not the end of the story, so this chapter explores dividends, stock values, and the connection between the two.

5.1 Definition and Example of a Bond

A *bond* is a certificate showing that a borrower owes a specified sum. To repay the money, the borrower has agreed to make interest and principal payments on designated dates. For example, imagine that Kreuger Enterprises just issued 100,000 bonds for \$1,000 each, where the bonds have a coupon rate of 5 percent and a maturity of two years. Interest on the bonds is to be paid yearly. This means that:

1. \$100 million ($= 100,000 \times \$1,000$) has been borrowed by the firm.
2. The firm must pay interest of \$5 million ($= 5\% \times \100 million) at the end of one year.
3. The firm must pay both \$5 million of interest and \$100 million of principal at the end of two years.

We now consider how to value a few different types of bonds.

5.2 How to Value Bonds

Pure Discount Bonds

The **pure discount bond** is perhaps the simplest kind of bond. It promises a single payment, say \$1, at a fixed future date. If the payment is one year from now, it is called a *one-year discount bond*; if it is two years from now, it is called a *two-year discount bond*, and so on.

Figure 5.1

Different Types of Bonds: C , Coupon Paid Every 6 Months; F , Face Value at Year 4 (maturity for pure discount and coupon bonds)

	Year 1		Year 2		Year 3		Year 4		...
Months	6	12	18	24	30	36	42	48	...
Pure discount bonds								F	
Coupon bonds	C	C	C	C	C	C	C	$F + C$	
Consols	C	C	C	C	C	C	C	C	C

The date when the issuer of the bond makes the last payment is called the **maturity date** of the bond, or just its *maturity* for short. The bond is said to mature or *expire* on the date of its final payment. The payment at maturity (\$1 in this example) is termed the bond's **face** or **par value**.

Pure discount bonds are often called *zero coupon bonds* or *zeros* to emphasize the fact that the holder receives no cash payments until maturity. We will use the terms *zero* and *discount* interchangeably to refer to bonds that pay no coupons.

The first row of Figure 5.1 shows the pattern of cash flows from a four-year pure discount bond. Note that the face value, F , is paid when the bond expires in the 48th month. There are no payments of either interest or principal prior to this date.

In the previous chapter, we indicated that one discounts a future cash flow to determine its present value. The present value of a pure discount bond can easily be determined by the techniques of the previous chapter. For short, we sometimes speak of the *value* of a bond instead of its present value.

Consider a pure discount bond that pays a face value of F in T years, where the interest rate is R in each of the T years. (We also refer to this rate as the *market interest rate*.) Because the face value is the only cash flow that the bond pays, the present value of this face amount is calculated as follows:

Value of a Pure Discount Bond:

$$PV = \frac{F}{(1 + R)^T}$$

The present value formula can produce some surprising results. Suppose that the interest rate is 10 percent. Consider a bond with a face value of \$1 million that matures in 20 years. Applying the formula to this bond, its PV is given by:

$$\begin{aligned} PV &= \frac{\$1 \text{ million}}{(1.1)^{20}} \\ &= \$148,644 \end{aligned}$$

or only about 15 percent of the face value.

Level Coupon Bonds

Typical bonds issued by either governments or corporations offer cash payments not just at maturity, but also at regular times in between. For example, payments on U.S. government issues and American corporate bonds are made every six months until the bonds mature.

These payments are called the **coupons** of the bond. The middle row of Figure 5.1 illustrates the case of a four-year, *level coupon bond*: The coupon, C , is paid every six months and is the same throughout the life of the bond.

Note that the face value of the bond, F , is paid at maturity (end of year 4). F is sometimes called the *principal* or the *denomination*. Bonds issued in the United States typically have face values of \$1,000, though this can vary with the type of bond.

As we mentioned before, the value of a bond is simply the present value of its cash flows. Therefore, the value of a level coupon bond is merely the present value of its stream of coupon payments plus the present value of its repayment of principal. Because a level coupon bond is just an annuity of C each period, together with a payment at maturity of \$1,000, the value of a level coupon bond is calculated as follows:

Value of a Level Coupon Bond:

$$PV = \frac{C}{1+R} + \frac{C}{(1+R)^2} + \cdots + \frac{C}{(1+R)^T} + \frac{\$1,000}{(1+R)^T}$$

where C is the coupon and the face value, F , is \$1,000. The value of the bond can be rewritten like this:

Value of a Level Coupon Bond:

$$PV = C \times A_R^T + \frac{\$1,000}{(1+R)^T}$$

As mentioned in the previous chapter, A_R^T is the present value of an annuity of \$1 per period for T periods at an interest rate per period of R .

EXAMPLE 5.1

Bond Prices Suppose it is November 2006 and we are considering a government bond. We see in *The Wall Street Journal* some *13s* of November 2010. This is jargon that means the annual coupon rate is 13 percent.¹ The face value is \$1,000, implying that the yearly coupon is \$130 ($= 13\% \times \$1,000$). Interest is paid each May and November, implying that the coupon every six months is \$65 ($= \$130/2$). The face value will be paid out in November 2010, four years from now. By this we mean that the purchaser obtains claims to the following cash flows:

5/07	11/07	5/08	11/08	5/09	11/09	5/10	11/10
\$65	\$65	\$65	\$65	\$65	\$65	\$65	\$65 + \$1,000

If the stated annual interest rate in the market is 10 percent per year, what is the present value of the bond?

Our work on compounding in the previous chapter showed that the interest rate over any six-month interval is half of the stated annual interest rate. In the current example, this semiannual rate is 5 percent ($= 10\%/2$). Because the coupon payment in each six-month period is \$65, and there are

(continued)

¹The coupon rate is specific to the bond. The coupon rate indicates what cash flow should appear in the numerator of the NPV equation. The coupon rate does *not* appear in the denominator of the NPV equation.

eight of these six-month periods from November 2006 to November 2010, the present value of the bond is:

$$\begin{aligned}
 PV &= \frac{\$65}{(1.05)} + \frac{\$65}{(1.05)^2} + \dots + \frac{\$65}{(1.05)^8} + \frac{\$1,000}{(1.05)^8} \\
 &= \$65 \times A_{0.05}^8 + \$1,000/(1.05)^8 \\
 &= (\$65 \times 6.463) + (\$1,000 \times 0.677) \\
 &= \$420.095 + \$677 \\
 &= \$1,097.095
 \end{aligned}$$

Traders will generally quote the bond as 109.7095,² indicating that it is selling at 109.7095 percent of the face value of \$1,000.

At this point, it is worthwhile to relate the preceding example of bond pricing to the discussion of compounding in the previous chapter. At that time, we distinguished between the stated annual interest rate and the effective annual interest rate. In particular, we pointed out that the effective annual interest rate is:

$$(1 + R/m)^m - 1$$

where R is the stated annual interest rate and m is the number of compounding intervals. Because $R = 10\%$ and $m = 2$ (the bond makes semiannual payments), the effective annual interest rate is

$$[1 + (0.10/2)]^2 - 1 = (1.05)^2 - 1 = 10.25\%$$

In other words, because the bond is paying interest twice a year, the bondholder earns a 10.25 percent return when compounding is considered.³

One final note concerning level coupon bonds: Although the preceding example concerns government bonds, corporate bonds are identical in form. For example, Du Pont Corporation has a 4.75 percent bond maturing in 2012. This means that Du Pont will make semiannual payments of \$23.75 ($=4.75\%/2 \times \$1,000$) between now and 2012 for each face value of \$1,000.

Consols

Not all bonds have a final maturity date. As we mentioned in the previous chapter, consols are bonds that never stop paying a coupon, have no final maturity date, and therefore never mature. Thus, a consol is a perpetuity. In the 18th century, the Bank of England issued such bonds, called “English consols.” These were bonds that the Bank of England guaranteed would pay the holder a cash flow forever! Through wars and depressions, the Bank of England continued to honor this commitment, and you can still buy such bonds in London today. The U.S. government also once sold consols to raise money to build the Panama Canal. Even though these U.S. bonds were supposed to last forever and to pay their coupons

²U.S. government bond prices are actually quoted in 32nds of a dollar, so a quote this precise would not be given for such bonds.

³For an excellent discussion of how to value semiannual payments, see J. T. Lindley, B. P. Helms, and M. Haddad, “A Measurement of the Errors in Intra-Period Compounding and Bond Valuation,” *The Financial Review* 22 (February 1987). We benefited from several conversations with the authors of this article.

forever, don't go looking for any. There is a special clause in the bond contract that gives the government the right to buy them back from the holders, and that is what the government has done. Such clauses are known as *call provisions*, and we study them later.

An important example of a consol, though, is called *preferred stock*. Preferred stock is stock that is issued by corporations and that provides the holder a fixed dividend in perpetuity. If there were never any question that the firm would actually pay the dividend on the preferred stock, such stock would in fact be a consol.

These instruments can be valued by the perpetuity formula of the previous chapter. For example, if the marketwide interest rate is 10 percent, a consol with a yearly interest payment of \$50 is valued at:

$$\frac{\$50}{0.10} = \$500$$

5.3 Bond Concepts

We complete our discussion of bonds by considering two concepts concerning them. First we examine the relationship between interest rates and bond prices. Then we define the concept of yield to maturity.

Interest Rates and Bond Prices

The discussion of level coupon bonds allows us to relate bond prices to interest rates. Consider the following example:

EXAMPLE 5.2

Bond Valuation The interest rate is 10 percent. A two-year bond with a 10 percent coupon pays interest of \$100 ($= \$1,000 \times 10\%$). For simplicity we assume that the interest is paid annually. In this case, we see that the bond is priced at its face value of \$1,000:

$$\$1,000 = \frac{\$100}{1.10} + \frac{\$1,000 + \$100}{(1.10)^2}$$

If the interest rate unexpectedly rises to 12 percent, the bond sells at:

$$\$966.20 = \frac{\$100}{1.12} + \frac{\$1,000 + \$100}{(1.12)^2}$$

Because \$966.20 is less than \$1,000, the bond is said to sell at a **discount**. This is a sensible result. Now that the interest rate is 12 percent, a newly issued bond with a 12 percent coupon rate will sell at \$1,000. This newly issued bond will have coupon payments of \$120 ($= 0.12 \times \$1,000$). Because our bond has interest payments of only \$100, investors will pay less than \$1,000 for it.

If interest rates fell to 8 percent, the bond would sell at:

$$\$1,035.67 = \frac{\$100}{1.08} + \frac{\$1,000 + \$100}{(1.08)^2}$$

Because \$1,035.67 is more than \$1,000, the bond is said to sell at a **premium**.

Thus, we find that bond prices fall with a rise in interest rates and rise with a fall in interest rates. Furthermore, the general principle is that a level coupon bond sells in the following ways:

1. At the face value of \$1,000 if the coupon rate is equal to the marketwide interest rate.
2. At a discount if the coupon rate is below the marketwide interest rate.
3. At a premium if the coupon rate is above the marketwide interest rate.

Yield to Maturity

Let's now consider the previous example *in reverse*. If our bond is selling at \$1,035.67, what return is a bondholder receiving? This can be answered by considering the following equation:

$$\$1,035.67 = \frac{\$100}{1 + y} + \frac{\$1,000 + \$100}{(1 + y)^2}$$

The unknown, y , is the discount rate that equates the price of the bond with the discounted value of the coupons and face value. Our earlier work implies that $y = 8\%$. Thus, traders state that the bond is yielding an 8 percent return. Bond traders also state that the bond has a **yield to maturity** of 8 percent. The yield to maturity is frequently called the bond's *yield* for short. So, we would say the bond with its 10 percent coupon is priced to yield 8 percent at \$1,035.67.

Bond Market Reporting

In 2002, data availability in the corporate bond market began to improve dramatically. Under new regulations, corporate bond dealers are now required to report trade information through what is known as the Transactions Report and Compliance Engine (TRACE). As this is written, transaction prices are now reported for more than 4,000 bonds. More bonds will be added over time.

The Present Value Formulas for Bonds

Pure Discount Bonds

$$PV = \frac{F}{(1 + R)^T}$$

Level Coupon Bonds

$$PV = C \left[\frac{1}{R} - \frac{1}{R \times (1 + R)^T} \right] + \frac{F}{(1 + R)^T} = C \times A_R^T + \frac{F}{(1 + R)^T}$$

where F is typically \$1,000 for a level coupon bond.

Consols

$$PV = \frac{C}{R}$$

Figure 5.2
Sample Wall Street Journal Bond Quotation

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Corporate Bonds

Thursday, January 19, 2006

Forty most active fixed-coupon corporate bonds

COMPANY (TICKER)	COUPON	MATURITY	LAST PRICE	LAST YIELD	*EST SPREAD	UST†	EST \$ VOL (000's)
Goldman Sachs Group Inc (GS)	4.500	Jun 15, 2010	97.984	5.016	71	5	116,790
JPMorgan Chase (JPM)	5.150	Oct 01, 2015	98.935	5.291	92	10	114,535
Wal-Mart Stores Inc (WMT)	5.250	Sep 01, 2035	97.286	5.435	89	30	106,785
Virginia Electric and Power Co (D)	6.000	Jan 15, 2036	101.113	5.920	138	30	102,050
Ameriprise Financial Inc (AMP)	5.650	Nov 15, 2015	101.670	5.427	105	10	78,419
Ameriprise Financial Inc (AMP)	5.350	Nov 15, 2010	101.107	5.086	78	5	77,720
AT&T Inc (SBC)	5.100	Sep 15, 2014	97.828	5.417	104	10	72,859
Telecom Italia Capital (TITIM)	6.375	Nov 15, 2033	100.195	6.359	181	30	71,279
Tyco International Group SA (TYC)	6.375	Oct 15, 2011	104.961	5.355	105	5	69,148
Marsh & McLennan Companies Inc (MMC)	5.150	Sep 15, 2010	99.773	5.204	90	5	68,850
COX Communications Inc (COXENT)	7.125	Oct 01, 2012	106.810	5.876	149	5	65,891
Mohawk Industries Inc (MHK)	6.125	Jan 15, 2016	101.159	5.969	159	10	65,400
Time Warner Inc (TWX)	7.700	May 01, 2032	113.181	6.632	209	30	63,575
iStar Financial Inc (SFI)	5.375	Apr 15, 2010	99.981	5.378	107	5	60,530
Kroger Co (KR)	5.500	Feb 01, 2013	98.542	5.755	138	10	58,626
Ohio Power Co (AEP)	5.500	Feb 15, 2013	101.661	5.215	84	10	55,150
Kroger Co (KR)	7.500	Apr 01, 2031	112.406	6.492	195	30	54,720
Toyota Motor Credit Corp (TOYOTA)	5.125	Jan 11, 2011	99.930	5.141	83	5	54,000
General Electric Capital Corp (GE)	5.875	Feb 15, 2012	104.601	4.985	61	5	53,751
Time Warner Inc (TWX)	6.875	May 01, 2012	106.527	5.623	124	5	53,436
Vale Overseas Ltd (VALE)	6.250	Jan 11, 2016	100.600	6.168	179	10	52,735
Morgan Stanley (MWD)	5.050	Jan 21, 2011	100.030	5.036	73	5	51,415
HSBC Finance Corp (HSBC)	4.625	Jan 15, 2008	99.514	4.885	53	2	51,115
Albertson's Inc (ABS)	8.000	May 01, 2031	99.093	8.083	353	30	50,549
United Technologies Corp (UTX)	7.125	Nov 15, 2010	109.487	4.885	58	5	50,420
Convergys Corp (CVG)	4.875	Dec 15, 2009	96.769	5.813	150	3	50,400
General Electric Capital Corp (GE)	6.125	Feb 22, 2011	105.465	4.895	59	5	49,912
Bear Stearns Companies Inc (BSC)	6.500	May 01, 2006	100.506	4.531	n.a.	n.a.	49,769
Wal-Mart Stores Inc (WMT)	4.550	May 01, 2013	97.800	4.912	54	10	49,352
Fortune Brands Inc (FO)	5.375	Jan 15, 2016	99.287	5.468	109	10	48,700
Lear Corp (LEA)	5.750	Aug 01, 2014	79.125	9.359	499	10	48,466
HSBC Finance Corp (HSBC)	5.500	Jan 19, 2016	100.418	5.445	107	10	48,195
Wells Fargo (WFC)	4.875	Jan 12, 2011	99.919	4.893	57	5	46,830
BellSouth Corp (BLS)	5.200	Sep 15, 2014	99.587	5.259	89	10	45,588
Johnson Controls Inc (JCI)	5.250	Jan 15, 2011	100.351	5.169	86	5	45,325
Comcast Corp (CMCSA)	4.950	Jun 15, 2016	93.867	5.741	137	10	44,279
GlaxoSmithKline Capital Inc (GSK)	4.375	Apr 15, 2014	96.153	4.949	58	10	43,249
WellPoint Inc (WLP)	5.850	Jan 15, 2036	101.016	5.778	123	30	42,600
Countrywide Home Loans Inc (CFC)	5.500	Feb 01, 2007	100.539	4.950	n.a.	n.a.	41,744
GlaxoSmithKline Capital Inc (GSK)	2.375	Apr 16, 2007	97.254	4.700	34	2	41,704
Time Warner Companies Inc (TWX)	9.150	Feb 01, 2023	122.903	6.850	230	30	41,109

Volume represents total volume for each issue; price/yield data are for trades of \$1 million and greater. * Estimated spreads, in basis points (100 basis points is one percentage point), over the 2, 3, 5, 10 or 30-year hot run Treasury note/bond. 2-year: 4.375 12/07; 3-year: 4.375 11/08; 5-year: 4.250 01/11; 10-year: 4.500 11/15; 30-year: 5.375 02/31. †Comparable U.S. Treasury issue.
 Source: MarketAxess Corporate BondTicker

To learn more about TRACE, visit www.nasdaq.com.

TRACE bond quotes are available at www.nasdaqbondinfo.com. We went to the site and entered “Deere” for the well-known manufacturer of green tractors. We found a total of eight bond issues outstanding. Here you can see the information we found for three of these:

Issue: DE.GG DEERE & CO. 7.125 03/03/2031								Time and Sales	Descriptive Data
In	Rating	Last Sale		Most Recent					
Portfolio	Moody's/S&P/Fitch	Date	Price	Yield	Date	Price	Yield		
<input type="checkbox"/>	A3 / A- / A	01/20/2006	122.333	5.476993	01/20/2006	122.33	5.476993		

Issue: DE.GA DEERE & COMPANY 8.95 06/15/2019								Time and Sales	Descriptive Data
In	Rating	Last Sale		Most Recent					
Portfolio	Moody's/S&P/Fitch	Date	Price	Yield	Date	Price	Yield		
<input type="checkbox"/>	A3 / A- / A	01/20/2006	113.50	4.599	01/20/2006	113.50	4.599		

Issue: DE.GB DEERE & COMPANY 8.50 01/09/2022								Time and Sales	Descriptive Data
In	Rating	Last Sale		Most Recent					
Portfolio	Moody's/S&P/Fitch	Date	Price	Yield	Date	Price	Yield		
<input type="checkbox"/>	A3 / A- / A	01/05/2006	132.115	5.463	01/05/2006	132.12	5.463		

Most of the information is self-explanatory. The price and yield columns show the price and yield to maturity of the most recent sales. Notice the last sale dates for the issue maturing in 2022. This bond had not had a reported trade for the last two weeks. A great feature of this Web site is the “Descriptive Data” link, which gives you more information about the bond issue such as call dates and coupon dates.

The Federal Reserve Bank of St. Louis maintains dozens of online files containing macroeconomic data as well as rates on U.S. Treasury issues. Go to www.stls.frb.org/fred/files.

As shown in Figure 5.2, *The Wall Street Journal* provides a daily snapshot of the data from TRACE by reporting the 40 most active issues. The information reported is again largely self-explanatory. The EST Spread is the estimated yield spread over a particular U.S. Treasury issue (a yield spread is just the difference in yields). The spread is reported in basis points, where 1 basis point is equal to .01 percent. The selected Treasury issue's maturity is given under UST, which is a standard abbreviation in the bond markets for U.S. Treasury. A "hot run" Treasury is the most recently issued of a particular maturity, better known as an on-the-run issue. Finally, the reported volume is the face value of bonds traded.

5.4 The Present Value of Common Stocks

Dividends versus Capital Gains

Our goal in this section is to value common stocks. We learned in the previous chapter that an asset's value is determined by the present value of its future cash flows. A stock provides two kinds of cash flows. First, stocks often pay dividends on a regular basis. Second, the stockholder receives the sale price when selling the stock. Thus, to value common stocks, we need to answer an interesting question. Which of the following is the value of a stock equal to?

1. The discounted present value of the sum of next period's dividend plus next period's stock price.
2. The discounted present value of all future dividends.

This is the kind of question that students would love to see on a multiple-choice exam: Both (1) and (2) are right.

To see that (1) and (2) are the same, let's start with an individual who will buy the stock and hold it for one year. In other words, she has a one-year *holding period*. In addition, she is willing to pay P_0 for the stock today. That is, she calculates:

$$P_0 = \frac{\text{Div}_1}{1 + R} + \frac{P_1}{1 + R} \quad (5.1)$$

Div_1 is the dividend paid at year's end, and P_1 is the price at year's end. P_0 is the PV of the common stock investment. The term in the denominator, R , is the appropriate discount rate for the stock.

That seems easy enough; but where does P_1 come from? P_1 is not pulled out of thin air. Rather, there must be a buyer at the end of year 1 who is willing to purchase the stock for P_1 . This buyer determines price as follows:

$$P_1 = \frac{\text{Div}_2}{1 + R} + \frac{P_2}{1 + R} \quad (5.2)$$

Substituting the value of P_1 from Equation 5.2 into Equation 5.1 yields:

$$\begin{aligned} P_0 &= \frac{1}{1 + R} \left[\text{Div}_1 + \left(\frac{\text{Div}_2 + P_2}{1 + R} \right) \right] \\ &= \frac{\text{Div}_1}{1 + R} + \frac{\text{Div}_2}{(1 + R)^2} + \frac{P_2}{(1 + R)^2} \end{aligned} \quad (5.3)$$

We can ask a similar question for Equation 5.3: Where does P_2 come from? An investor at the end of year 2 is willing to pay P_2 because of the dividend and stock price at year 3. This process can be repeated *ad nauseam*.⁴ At the end, we are left with this:

$$P_0 = \frac{\text{Div}_1}{1 + R} + \frac{\text{Div}_2}{(1 + R)^2} + \frac{\text{Div}_3}{(1 + R)^3} + \dots = \sum_{t=1}^{\infty} \frac{\text{Div}_t}{(1 + R)^t} \tag{5.4}$$

Thus, the value of a firm’s common stock to the investor is equal to the present value of all of the expected future dividends.

This is a very useful result. A common objection to applying present value analysis to stocks is that investors are too shortsighted to care about the long-run stream of dividends. These critics argue that an investor will generally not look past his or her time horizon. Thus, prices in a market dominated by short-term investors will reflect only near-term dividends. However, our discussion shows that a long-run dividend discount model holds even when investors have short-term time horizons. Although an investor may want to cash out early, she must find another investor who is willing to buy. The price this second investor pays is dependent on dividends *after* his date of purchase.

Valuation of Different Types of Stocks

The preceding discussion shows that the value of the firm is the present value of its future dividends. How do we apply this idea in practice? Equation 5.4 represents a very general model and is applicable regardless of whether the level of expected dividends is growing, fluctuating, or constant. The general model can be simplified if the firm’s dividends are expected to follow some basic patterns: (1) zero growth, (2) constant growth, and (3) differential growth. These cases are illustrated in Figure 5.3.

Case 1 (Zero Growth) The value of a stock with a constant dividend is given by

$$P_0 = \frac{\text{Div}_1}{1 + R} + \frac{\text{Div}_2}{(1 + R)^2} + \dots = \frac{\text{Div}_1}{R}$$

Here it is assumed that $\text{Div}_1 = \text{Div}_2 = \dots = \text{Div}$. This is just an application of the perpetuity formula from a previous chapter.

Case 2 (Constant Growth) Dividends grow at rate g , as follows:

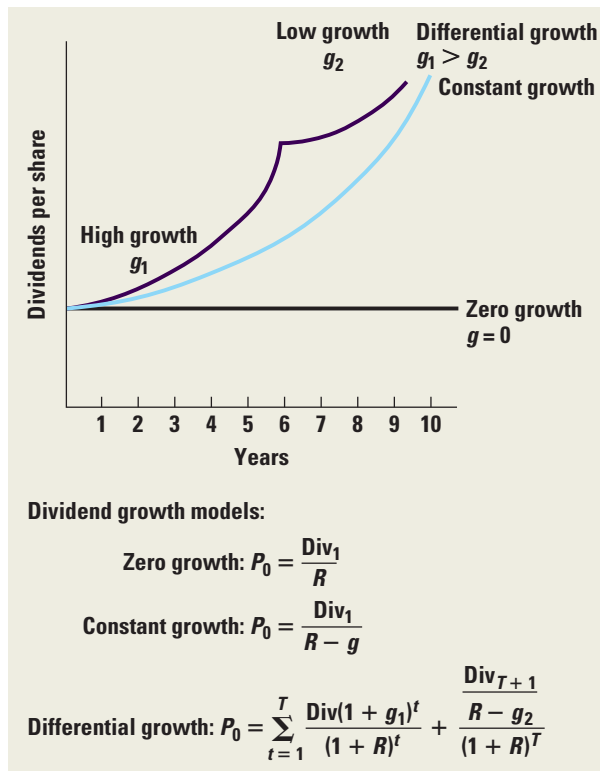
End of Year Dividend:				
1	2	3	4	...
Div_1	$\text{Div}_1(1 + g)$	$\text{Div}_1(1 + g)^2$	$\text{Div}_1(1 + g)^3$	

Note that Div_1 is the dividend at the end of the *first* period.

⁴This procedure reminds us of the physicist lecturing about the origins of the universe. He was approached by an elderly gentleman in the audience who disagreed with the lecture. The attendee said that the universe rests on the back of a huge turtle. When the physicist asked what the turtle rested on, the gentleman said another turtle. Anticipating the physicist’s objections, the attendee said, “Don’t tire yourself out, young fellow. It’s turtles all the way down.”

Figure 5.3

Zero Growth, Constant Growth, and Differential Growth Patterns

**EXAMPLE 5.3**

Projected Dividends Hampshire Products will pay a dividend of \$4 per share a year from now. Financial analysts believe that dividends will rise at 6 percent per year for the foreseeable future. What is the dividend per share at the end of each of the first five years? With 6 percent growth we have this:

End of Year Dividend:

1	2	3	4	5
\$4.00	$\$4 \times (1.06)$	$\$4 \times (1.06)^2$	$\$4 \times (1.06)^3$	$\$4 \times (1.06)^4$
	= \$4.24	= \$4.4944	= \$4.7641	= \$5.0499

The value of a common stock with dividends growing at a constant rate is

$$P_0 = \frac{\text{Div}_1}{1 + R} + \frac{\text{Div}_1 (1 + g)}{(1 + R)^2} + \frac{\text{Div}_1 (1 + g)^2}{(1 + R)^3} + \frac{\text{Div}_1 (1 + g)^3}{(1 + R)^4} + \dots = \frac{\text{Div}_1}{R - g}$$

where g is the growth rate. Div_1 is the dividend on the stock at the end of the first period. This is the formula for the present value of a growing perpetuity, which we derived in a previous chapter.

EXAMPLE 5.4

Stock Valuation Suppose an investor is considering the purchase of a share of the Utah Mining Company. The stock will pay a \$3 dividend a year from today. This dividend is expected to grow at 10 percent per year ($g = 10\%$) for the foreseeable future. The investor thinks that the required return (R) on this stock is 15 percent, given her assessment of Utah Mining's risk. (We also refer to R as the discount rate for the stock.) What is the value of a share of Utah Mining Company's stock?

Using the constant growth formula of case 2, we assess the value to be \$60:

$$\$60 = \frac{\$3}{.15 - .10}$$

(continued)

P_0 is quite dependent on the value of g . If g had been estimated to be 12.5 percent, the value of the share would have been:

$$\$120 = \frac{\$3}{.15 - .125}$$

The stock price doubles (from \$60 to \$120) when g increases only 25 percent (from 10 percent to 12.5 percent). Because of P_0 's dependence on g , one must maintain a healthy sense of skepticism when using this constant growth of dividends model.

Furthermore, note that P_0 is equal to infinity when the growth rate, g , equals the discount rate, R . Because stock prices do not grow infinitely, an estimate of g greater than R implies an error in estimation. More will be said about this point later.

Case 3 (Differential Growth) In this case, an algebraic formula would be too unwieldy. Instead we present examples.

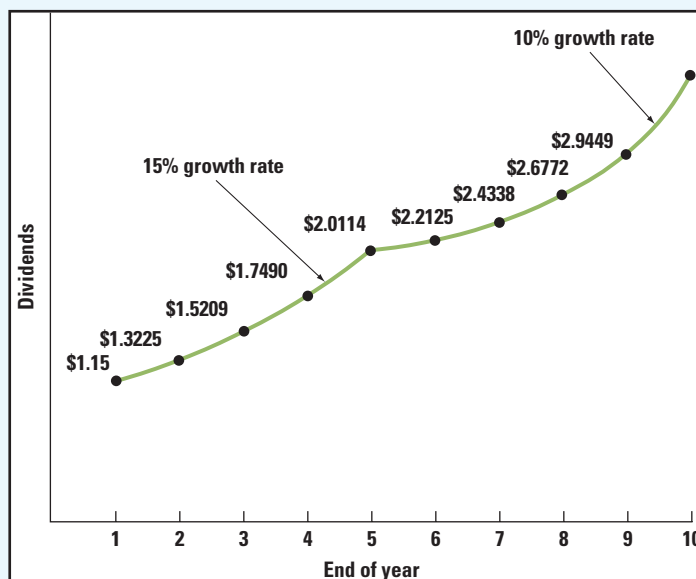
EXAMPLE 5.5

Differential Growth Consider the stock of Elixir Drug Company, which has a new massage ointment and is enjoying rapid growth. The dividend for a share of stock a year from today will be \$1.15. During the following four years the dividend will grow at 15 percent per year ($g_1 = 15\%$). After that, growth (g_2) will equal 10 percent per year. Can you calculate the present value of the stock if the required return (R) is 15 percent?

Figure 5.4 displays the growth in the dividends. We need to apply a two-step process to discount these dividends. We first calculate the net present value of the dividends growing at 15 percent per annum. That is, we first calculate the present value of the dividends at the end of each of the first five years. Second, we calculate the present value of the dividends that begin at the end of year 6.

Figure 5.4

Growth in Dividends for Elixir Drug Company



(continued)

Calculate Present Value of First Five Dividends The present value of dividend payments in years 1 through 5 is as follows:

Future Year	Growth Rate (g_1)	Expected Dividend	Present Value
1	.15	\$1.15	\$1
2	.15	1.3225	1
3	.15	1.5209	1
4	.15	1.7490	1
5	.15	2.0114	1
Years 1–5	The present value of dividends = \$5		

The growing annuity formula of the previous chapter could normally be used in this step. However, note that dividends grow at 15 percent, which is also the discount rate. Because $g = R$, the growing annuity formula cannot be used in this example.

Calculate Present Value of Dividends Beginning at End of Year 6 This is the procedure for deferred perpetuities and deferred annuities that we mentioned in the previous chapter. The dividends beginning at the end of year 6 are as follows:

End of Year Dividend:

6	7	8	9
$\text{Div}_5 \times (1 + g_2)$	$\text{Div}_5 \times (1 + g_2)^2$	$\text{Div}_5 \times (1 + g_2)^3$	$\text{Div}_5 \times (1 + g_2)^4$
$\$2.0114 \times 1.10$	$\$2.0114 \times (1.10)^2$	$\$2.0114 \times (1.10)^3$	$\$2.0114 \times (1.10)^4$
= \$2.2125	= \$2.4338	= \$2.6772	= \$2.9449

As stated in the previous chapter, the growing perpetuity formula calculates present value as of one year prior to the first payment. Because the payment begins at the end of year 6, the present value formula calculates present value as of the end of year 5.

The price at the end of year 5 is given by

$$P_5 = \frac{\text{Div}_6}{R - g_2} = \frac{\$2.2125}{.15 - .10} = \$44.25$$

The present value of P_5 today is

$$\frac{P_5}{(1 + R)^5} = \frac{\$44.25}{(1.15)^5} = \$22$$

The present value of *all* dividends today is \$27 (= \$22 + 5).

5.5 Estimates of Parameters in the Dividend Growth Model

The value of the firm is a function of its growth rate, g , and its discount rate, R . How do we estimate these variables?

Where Does g Come From?

The previous discussion of stocks assumed that dividends grow at the rate g . We now want to estimate this rate of growth. This section extends the discussion of growth contained in Chapter 3. Consider a business whose earnings next year are expected to be the same as earnings this year unless a *net investment* is made. This situation is likely to occur because net investment is equal to gross, or total, investment less depreciation. A net investment of zero occurs when *total investment* equals depreciation. If total investment is equal to depreciation, the firm's physical plant is maintained, consistent with no growth in earnings.

Net investment will be positive only if some earnings are not paid out as dividends—that is, only if some earnings are retained.⁵ This leads to the following equation:

$$\begin{array}{r} \text{Earnings} \\ \text{next} \\ \text{year} \end{array} = \begin{array}{r} \text{Earnings} \\ \text{this} \\ \text{year} \end{array} + \underbrace{\begin{array}{r} \text{Retained} \\ \text{earnings} \\ \text{this year} \end{array} \times \begin{array}{r} \text{Return on} \\ \text{retained} \\ \text{earnings} \end{array}}_{\text{Increase in earnings}} \quad (5.5)$$

The increase in earnings is a function of both the *retained earnings* and the *return on the retained earnings*.

We now divide both sides of Equation 5.5 by earnings this year, yielding

$$\frac{\text{Earnings next year}}{\text{Earnings this year}} = \frac{\text{Earnings this year}}{\text{Earnings this year}} + \left(\frac{\text{Retained earnings this year}}{\text{Earnings this year}} \right) \times \text{Return on retained earnings} \quad (5.6)$$

The left side of Equation 5.6 is simply 1 plus the growth rate in earnings, which we write as $1 + g$. The ratio of retained earnings to earnings is called the **retention ratio**. Thus we can write

$$1 + g = 1 + \text{Retention ratio} \times \text{Return on retained earnings} \quad (5.7)$$

It is difficult for a financial analyst to determine the return to be expected on currently retained earnings: The details on forthcoming projects are not generally public information. However, it is frequently assumed that the projects selected in the current year have an anticipated return equal to returns from projects in other years. Here we can estimate the anticipated return on current retained earnings by the historical **return on equity** or ROE. After all, ROE is simply the return on the firm's entire equity, which is the return on the cumulation of all the firm's past projects.

From Equation 5.7, we have a simple way to estimate growth:

Formula for Firm's Growth Rate:

$$g = \text{Retention ratio} \times \text{Return on retained earnings} \quad (5.8)$$

Previously, g referred to growth in dividends. However, the growth in earnings is equal to the growth rate in dividends in this context, because as we will presently see, the ratio of dividends to earnings is held constant. In fact, as you have probably figured out, g is the sustainable growth rate we introduced in Chapter 3.

⁵We ignore the possibility of the issuance of stocks or bonds to raise capital. These possibilities are considered in later chapters.

EXAMPLE 5.6

Earnings Growth Pagemaster Enterprises just reported earnings of \$2 million. It plans to retain 40 percent of its earnings. The historical return on equity (ROE) has been 16 percent, a figure that is expected to continue into the future. How much will earnings grow over the coming year?

We first perform the calculation without reference to Equation 5.8. Then, we use Equation 5.8 as a check.

Calculation without Reference to Equation 5.8 The firm will retain \$800,000 ($= 40\% \times \2 million). Assuming that historical ROE is an appropriate estimate for future returns, the anticipated increase in earnings is:

$$\$800,000 \times .16 = \$128,000$$

The percentage growth in earnings is:

$$\frac{\text{Change in earnings}}{\text{Total earnings}} = \frac{\$128,000}{\$2 \text{ million}} = .064$$

This implies that earnings in one year will be \$2,128,000 ($= \$2,000,000 \times 1.064$).

Check Using Equation 5.8 We use $g = \text{Retention ratio} \times \text{ROE}$. We have:

$$g = .4 \times .16 = .064$$

Where Does R Come From?

Thus far, we have taken the required return, or discount rate R , as given. We will have quite a bit to say about this subject in later chapters. For now, we want to examine the implications of the dividend growth model for this required return. Earlier we calculated P_0 as follows:

$$P_0 = \text{Div}_1 / (R - g)$$

If we rearrange this to solve for R , we get:

$$\begin{aligned} R - g &= \text{Div}_1 / P_0 \\ R &= \text{Div}_1 / P_0 + g \end{aligned} \tag{5.9}$$

This tells us that the total return, R , has two components. The first of these, Div_1 / P_0 , is called the **dividend yield**. Because this is calculated as the expected cash dividend divided by the current price, it is conceptually similar to the current yield on a bond, which is the annual coupon divided by the bond's price.

The second part of the total return is the growth rate, g . As we will verify shortly, the dividend growth rate is also the rate at which the stock price grows. Thus, this growth rate can be interpreted as the **capital gains yield**—that is, the rate at which the value of the investment grows.

To illustrate the components of the required return, suppose we observe a stock selling for \$20 per share. The next dividend will be \$1 per share. You think that the dividend will grow by 10 percent per year more or less indefinitely. What return does this stock offer you if this is correct?

The dividend growth model calculates total return as:

$$\begin{aligned} R &= \text{Dividend yield} + \text{Capital gains yield} \\ R &= \text{Div}_1 / P_0 + g \end{aligned}$$

In this case, total return works out to be:

$$\begin{aligned} R &= \$1/20 + 10\% \\ &= 5\% + 10\% \\ &= 15\% \end{aligned}$$

This stock, therefore, has an expected return of 15 percent.

We can verify this answer by calculating the price in one year, P_1 , using 15 percent as the required return. Based on the dividend growth model, this price is:

$$\begin{aligned} P_1 &= \text{Div}_1 \times (1 + g) / (R - g) \\ &= \$1 \times 1.10 / (.15 - .10) \\ &= \$1.10 / .05 \\ &= \$22 \end{aligned}$$

Notice that this \$22 is $\$20 \times 1.1$, so the stock price has grown by 10 percent as it should. If you pay \$20 for the stock today, you will get a \$1 dividend at the end of the year, and you will have a $\$22 - 20 = \2 gain. Your dividend yield is thus $\$1/20 = 5$ percent. Your capital gains yield is $\$2/20 = 10$ percent, so your total return would be 5 percent + 10 percent = 15 percent.

To get a feel for actual numbers in this context, consider that, according to the 2006 Value Line *Investment Survey*, Procter & Gamble's dividends were expected to grow by 9.5 percent over the next 5 or so years, compared to a historical growth rate of 10.5 percent over the preceding 5 years and 11.5 percent over the preceding 10 years. In 2006, the projected dividend for the coming year was given as \$1.12. The stock price at that time was about \$58 per share. What is the return investors require on P&G? Here the dividend yield is 1.9 percent and the capital gains yield is 9.5 percent, giving a total required return of 11.4 percent on P&G stock.

EXAMPLE 5.7

Calculating the Required Return Pagemaster Enterprises, the company examined in the previous example, has 1,000,000 shares of stock outstanding. The stock is selling at \$10. What is the required return on the stock?

Because the retention ratio is 40 percent, the **payout ratio** is 60 percent ($= 1 - \text{retention ratio}$). The payout ratio is the ratio of dividends/earnings. Because earnings a year from now will be \$2,128,000 ($= \$2,000,000 \times 1.064$), dividends will be \$1,276,800 ($= .60 \times \$2,128,000$). Dividends per share will be \$1.28 ($= \$1,276,800/1,000,000$). Given our previous result that $g = .064$, we calculate R from (5.9) as follows:

$$.192 = \frac{\$1.28}{10.00} + .064$$

A Healthy Sense of Skepticism

It is important to emphasize that our approach merely *estimates* g ; our approach does not *determine* g precisely. We mentioned earlier that our estimate of g is based on a number of assumptions. For example, we assume that the return on reinvestment of future retained earnings is equal to the firm's past ROE. We assume that the future retention ratio is equal to the past retention ratio. Our estimate for g will be off if these assumptions prove to be wrong.

Unfortunately, the determination of R is highly dependent on g . For example, if g is estimated to be 0 in our example, R equals 12.8 percent ($= \$1.28/\10.00). If g is estimated to be 12 percent, R equals 24.8 percent ($= \$1.28/\$10.00 + 12\%$). Thus, one should view estimates of R with a healthy sense of skepticism.

Because of the preceding, some financial economists generally argue that the estimation error for R or a single security is too large to be practical. Therefore, they suggest calculating the average R for an entire industry. This R would then be used to discount the dividends of a particular stock in the same industry.

One should be particularly skeptical of two polar cases when estimating R for individual securities. First, consider a firm currently paying no dividend. The stock price will be above zero because investors believe that the firm may initiate a dividend at some point or the firm may be acquired at some point. However, when a firm goes from no dividends to a positive number of dividends, the implied growth rate is *infinite*. Thus, Equation 5.9 must be used with extreme caution here, if at all—a point we emphasize later in this chapter.

Second, we mentioned earlier that the value of the firm is infinite when g is equal to R . Because prices for stocks do not grow infinitely, an analyst whose estimate of g for a particular firm is equal to or above R must have made a mistake. Most likely the analyst's high estimate for g is correct for the next few years. However, firms simply cannot maintain an abnormally high growth rate *forever*. The analyst's error was to use a short-run estimate of g in a model requiring a perpetual growth rate.

5.6 Growth Opportunities

We previously spoke of the growth rate of dividends. We now want to address the related concept of growth opportunities. Imagine a company with a level stream of earnings per share in perpetuity. The company pays all of these earnings out to stockholders as dividends. Hence we have:

$$\text{EPS} = \text{Div}$$

where EPS is *earnings per share* and Div is dividends per share. A company of this type is frequently called a *cash cow*.

The perpetuity formula of the previous chapter gives the value of a share of stock:

Value of a Share of Stock When a Firm Acts as a Cash Cow:

$$\frac{\text{EPS}}{R} = \frac{\text{Div}}{R}$$

where R is the discount rate on the firm's stock.

This policy of paying out all earnings as dividends may not be the optimal one. Many firms have *growth opportunities*: opportunities to invest in profitable projects. Because these projects can represent a significant fraction of the firm's value, it would be foolish to forgo them in order to pay out all earnings as dividends.

Although firms frequently think in terms of a *set* of growth opportunities, let's focus on only one opportunity—that is, the opportunity to invest in a single project. Suppose the firm retains the entire dividend at date 1 to invest in a particular capital budgeting project. The net present value *per share* of the project as of date 0 is *NPVGO*, which stands for the *net present value (per share) of the growth opportunity*.

What is the price of a share of stock at date 0 if the firm decides to take on the project at date 1? Because the per share value of the project is added to the original stock price, the stock price must now be this:

Stock Price after Firm Commits to New Project:

$$\frac{\text{EPS}}{R} + \text{NPVGO} \tag{5.10}$$

Thus Equation 5.10 indicates that the price of a share of stock can be viewed as the sum of two different items. The first term (EPS/R) is the value of the firm if it rested on its laurels—that is, if it simply distributed all earnings to the stockholders. The second term is the *additional* value if the firm retains earnings to fund new projects.

EXAMPLE 5.8

Growth Opportunities Sarro Shipping, Inc., expects to earn \$1 million per year in perpetuity if it undertakes no new investment opportunities. There are 100,000 shares of stock outstanding, so earnings per share equal \$10 ($= \$1,000,000/100,000$). The firm will have an opportunity at date 1 to spend \$1,000,000 on a new marketing campaign. The new campaign will increase earnings in every subsequent period by \$210,000 (or \$2.10 per share). This is a 21 percent return per year on the project. The firm's discount rate is 10 percent. What is the value per share before and after deciding to accept the marketing campaign?

The value of a share of Sarro Shipping before the campaign is

Value of a Share of Sarro When Firm Acts as a Cash Cow:

$$\frac{EPS}{R} = \frac{\$10}{.1} = \$100$$

The value of the marketing campaign as of date 1 is

Value of Marketing Campaign at Date 1:

$$-\$1,000,000 + \frac{\$210,000}{.1} = \$1,100,000 \quad (5.11)$$

Because the investment is made at date 1 and the first cash inflow occurs at date 2, Equation 5.11 represents the value of the marketing campaign as of date 1. We determine the value at date 0 by discounting back one period as follows:

Value of Marketing Campaign at Date 0:

$$\frac{\$1,100,000}{1.1} = \$1,000,000$$

Thus NPVGO per share is \$10 ($= \$1,000,000/100,000$).

The price per share is

$$EPS/R + NPVGO = \$100 + 10 = \$110$$

The calculation in our example can also be made on a straight net present value basis. Because all the earnings at date 1 are spent on the marketing effort, no dividends are paid to stockholders at that date. Dividends in all subsequent periods are \$1,210,000 ($= \$1,000,000 + \$210,000$). In this case \$1,000,000 is the annual dividend when Sarro is a cash cow. The additional contribution to the dividend from the marketing effort is \$210,000. Dividends per share are \$12.10 ($= \$1,210,000/100,000$). Because these dividends start at date 2, the price per share at date 1 is \$121 ($= \$12.10/.1$). The price per share at date 0 is \$110 ($= \$121/1.1$).

Note that value is created in this example because the project earned a 21 percent rate of return when the discount rate was only 10 percent. No value would have been created had the project earned a 10 percent rate of return. The NPVGO would have been zero, and value would have been negative had the project earned a percentage return below 10 percent. The NPVGO would be negative in that case.

Two conditions must be met in order to increase value:

1. Earnings must be retained so that projects can be funded.⁶
2. The projects must have positive net present value.

Surprisingly, a number of companies seem to invest in projects known to have *negative* net present values. For example, in the late 1970s, oil companies and tobacco companies were flush with cash. Due to declining markets in both industries, high dividends and low investment would have been the rational action. Unfortunately, a number of companies in both industries reinvested heavily in what were widely perceived to be negative NPVGO projects.

Given that NPV analysis (such as that presented in the previous chapter) is common knowledge in business, why would managers choose projects with negative NPVs? One conjecture is that some managers enjoy controlling a large company. Because paying dividends in lieu of reinvesting earnings reduces the size of the firm, some managers find it emotionally difficult to pay high dividends.

Growth in Earnings and Dividends versus Growth Opportunities

As mentioned earlier, a firm's value increases when it invests in growth opportunities with positive NPVGOs. A firm's value falls when it selects opportunities with negative NPVGOs. However, dividends grow whether projects with positive NPVs or negative NPVs are selected. This surprising result can be explained by the following example.

EXAMPLE 5.9

NPV versus Dividends Lane Supermarkets, a new firm, will earn \$100,000 a year in perpetuity if it pays out all its earnings as dividends. However, the firm plans to invest 20 percent of its earnings in projects that earn 10 percent per year. The discount rate is 18 percent. An earlier formula tells us that the growth rate of dividends is:

$$g = \text{Retention ratio} \times \text{Return on retained earnings} = .2 \times .10 = 2\%$$

For example, in this first year of the new policy, dividends are \$80,000 [= $(1 - .2) \times \$100,000$]. Dividends next year are \$81,600 (= $\$80,000 \times 1.02$). Dividends the following year are \$83,232 [= $\$80,000 \times (1.02)^2$] and so on. Because dividends represent a fixed percentage of earnings, earnings must grow at 2 percent a year as well.

However, note that the policy reduces value because the rate of return on the projects of 10 percent is less than the discount rate of 18 percent. That is, the firm would have had a higher value at date 0 if it had a policy of paying all its earnings out as dividends. Thus, a policy of investing in projects with negative NPVs rather than paying out earnings as dividends will lead to growth in dividends and earnings, but will reduce value.

Dividends or Earnings: Which to Discount?

As mentioned earlier, this chapter applied the growing perpetuity formula to the valuation of stocks. In our application, we discounted dividends, not earnings. This is sensible because investors select a stock for what they can get out of it. They get only two things out of a stock: dividends and the ultimate sale price, which is determined by what future investors expect to receive in dividends.

⁶Later in the text, we speak of issuing stock or debt to fund projects.

The calculated stock price would be too high were earnings to be discounted instead of dividends. As we saw in our estimation of a firm's growth rate, only a portion of earnings goes to the stockholders as dividends. The remainder is retained to generate future dividends. In our model, retained earnings are equal to the firm's investment. To discount earnings instead of dividends would be to ignore the investment a firm must make today to generate future returns.

The No-Dividend Firm

Students frequently ask the following questions: If the dividend discount model is correct, why aren't no-dividend stocks selling at zero? This is a good question and gets at the goals of the firm. A firm with many growth opportunities faces a dilemma. The firm can pay out dividends now, or it can forgo dividends now so that it can make investments that will generate even greater dividends in the future.⁷ This is often a painful choice because a strategy of dividend deferment may be optimal yet unpopular among certain stockholders.

Many firms choose to pay no dividends—and these firms sell at positive prices. For example, most Internet firms, such as Amazon.com, Google, and eBay, pay no dividends. Rational shareholders believe that either they will receive dividends at some point or they will receive something just as good. That is, the firm will be acquired in a merger, with the stockholders receiving either cash or shares of stock at that time.

Of course, the actual application of the dividend discount model is difficult for firms of this type. Clearly the model for constant growth of dividends does not apply. Though the differential growth model can work in theory, the difficulties of estimating the date of first dividend, the growth rate of dividends after that date, and the ultimate merger price make application of the model quite difficult in reality.

Empirical evidence suggests that firms with high growth rates are likely to pay lower dividends, a result consistent with the analysis here. For example, consider McDonald's Corporation. The company started in the 1950s and grew rapidly for many years. It paid its first dividend in 1975, though it was a billion-dollar company (in both sales and market value of stockholders' equity) prior to that date. Why did it wait so long to pay a dividend? It waited because it had so many positive growth opportunities (additional locations for new hamburger outlets) to take advantage of.

5.7 The Dividend Growth Model and the NPVGO Model

This chapter has revealed that the price of a share of stock is the sum of its price as a cash cow plus the per-share value of its growth opportunities. The Sarro Shipping example illustrated this formula using only one growth opportunity. We also used the growing perpetuity formula to price a stock with a steady growth in dividends. When the formula is applied to stocks, it is typically called the *dividend growth model*. A steady growth in dividends results from a continual investment in growth opportunities, not just investment in a single opportunity. Therefore, it is worthwhile to compare the dividend growth model with the *NPVGO model* when growth occurs through continual investing.

We can use an example to illustrate the main points. Suppose Cumberland Book Publishers has EPS of \$10 at the end of the first year, a dividend payout ratio of 40 percent, a

⁷A third alternative is to issue stock so the firm has enough cash both to pay dividends and to invest. This possibility is explored in a later chapter.

discount rate of 16 percent, and a return on its retained earnings of 20 percent. Because the firm retains some of its earnings each year, it is selecting growth opportunities each year. This is different from Sarro Shipping, which had a growth opportunity in only one year. We wish to calculate the price per share using both the dividend growth model and the NPVGO model.

The Dividend Growth Model

The dividends at date 1 are $.40 \times \$10 = \4 per share. The retention ratio is $.60$ ($1 - .40$), implying a growth rate in dividends of $.12$ ($= .60 \times .20$).

From the dividend growth model, the price of a share of stock today is

$$\frac{\text{Div}_1}{R - g} = \frac{\$4}{.16 - .12} = \$100$$

The NPVGO Model

Using the NPVGO model, it is more difficult to value a firm with growth opportunities each year (like Cumberland) than a firm with growth opportunities in only one year (like Sarro). To value according to the NPVGO model, we need to calculate on a per-share basis (1) the net present value of a single growth opportunity, (2) the net present value of all growth opportunities, and (3) the stock price if the firm acts as a cash cow—that is, the value of the firm without these growth opportunities. The value of the firm is the sum of (2) + (3).

1. *Value per share of a single growth opportunity:* Out of the earnings per share of \$10 at date 1, the firm retains \$6 ($= .6 \times \10) at that date. The firm earns \$1.20 ($= \$6 \times .20$) per year in perpetuity on that \$6 investment. The NPV from the investment is calculated as follows:

Per-Share NPV Generated from Investment of Date 1:

$$-\$6 + \frac{\$1.20}{.16} = \$1.50 \quad (5.12)$$

That is, the firm invests \$6 to reap \$1.20 per year on the investment. The earnings are discounted at 16 percent, implying a value per share from the project of \$1.50. Because the investment occurs at date 1 and the first cash flow occurs at date 2, \$1.50 is the value of the investment at *date 1*. In other words, the NPV from the date 1 investment has *not* yet been brought back to date 0.

2. *Value per share of all opportunities:* As pointed out earlier, the growth rate of earnings and dividends is 12 percent. Because retained earnings are a fixed percentage of total earnings, retained earnings must also grow at 12 percent a year. That is, retained earnings at date 2 are \$6.72 ($= \6×1.12), retained earnings at date 3 are \$7.5264 [$= \$6 \times (1.12)^2$], and so on.

Let's analyze the retained earnings at date 2 in more detail. Because projects will always earn 20 percent per year, the firm earns \$1.344 ($= \$6.72 \times .20$) in each future year on the \$6.72 investment at date 2.

Here is the NPV from the investment:

NPV per Share Generated from Investment at Date 2:

$$-\$6.72 + \frac{\$1.344}{.16} = \$1.68 \quad (5.13)$$

\$1.68 is the NPV as of date 2 of the investment made at date 2. The NPV from the date 2 investment has *not* yet been brought back to date 0.

Now consider the retained earnings at date 3 in more detail. The firm earns \$1.5053 ($= \$7.5264 \times .20$) per year on the investment of \$7.5264 at date 3.

The NPV from the investment is thus:

NPV per Share Generated from Investment at Date 3:

$$-\$7.5264 + \frac{\$1.5053}{.16} = \$1.882 \quad (5.14)$$

From Equations 5.12, 5.13, and 5.14, the NPV per share of all of the growth opportunities, discounted back to date 0, is:

$$\frac{\$1.50}{1.16} + \frac{\$1.68}{(1.16)^2} + \frac{\$1.882}{(1.16)^3} + \dots \quad (5.15)$$

Because it has an infinite number of terms, this expression looks quite difficult to compute. However, there is an easy simplification. Note that retained earnings are growing at 12 percent per year. Because all projects earn the same rate of return per year, the NPVs in Equations 5.12, 5.13, and 5.14 are also growing at 12 percent per year. Hence, we can write Equation 5.15 as:

$$\frac{\$1.50}{1.16} + \frac{\$1.50 \times 1.12}{(1.16)^2} + \frac{\$1.50 \times (1.12)^2}{(1.16)^3} + \dots$$

This is a growth perpetuity whose value is:

$$\text{NPVGO} = \$ \frac{1.50}{.16 - .12} = \$37.50$$

Because the first NPV of \$1.50 occurs at date 1, the NPVGO is \$37.50 as of date 0. In other words, the firm's policy of investing in new projects from retained earnings has an NPV of \$37.50.

3. *Value per share if the firm is a cash cow:* We now assume that the firm pays out all of its earnings as dividends. The dividends would be \$10 per year in this case. Because there would be no growth, the value per share would be evaluated by the perpetuity formula:

$$\frac{\text{Div}}{R} = \frac{\$10}{.16} = \$62.50$$

Summation

Equation 5.10 states that value per share is the value of a cash cow plus the value of the growth opportunities. This is

$$\$100 = \$62.50 + 37.50$$

Hence, value is the same whether calculated by a discounted dividend approach or a growth opportunities approach. The share prices from the two approaches must be equal because the approaches are different yet equivalent methods of applying concepts of present value.

5.8 Price–Earnings Ratio

We argued earlier that one should not discount earnings to determine price per share. Nevertheless, financial analysts frequently relate earnings and price per share, as made evident by their heavy reliance on the price–earnings (or PE) ratio.

Our previous discussion stated that:

$$\text{Price per share} = \frac{\text{EPS}}{R} + \text{NPVGO}$$

Dividing by EPS yields:

$$\frac{\text{Price per share}}{\text{EPS}} = \frac{1}{R} + \frac{\text{NPVGO}}{\text{EPS}}$$

The left side is the formula for the price–earnings ratio. The equation shows that the PE ratio is related to the net present value of growth opportunities. As an example, consider two firms, each having just reported earnings per share of \$1. However, one firm has many valuable growth opportunities, whereas the other firm has no growth opportunities at all. The firm with growth opportunities should sell at a higher price because an investor is buying both current income of \$1 and growth opportunities. Suppose that the firm with growth opportunities sells for \$16 and the other firm sells for \$8. The \$1 earnings per share number appears in the denominator of the PE ratio for both firms. Thus, the PE ratio is 16 for the firm with growth opportunities but only 8 for the firm without the opportunities.

This explanation seems to hold fairly well in the real world. Electronic and other high-tech stocks generally sell at very high PE ratios (or *multiples*, as they are often called) because they are perceived to have high growth rates. In fact, some technology stocks sell at high prices even though the companies have never earned a profit. Conversely, railroads, utilities, and steel companies sell at lower multiples because of the prospects of lower growth. Table 5.1 contains PE ratios in 2006 for some well-known companies and the S&P 500 Index. Notice the variation across industries.

Of course, the market is merely pricing *perceptions* of the future, not the future itself. We will argue later in the text that the stock market generally has realistic perceptions of a firm's prospects. However, this is not always true. In the late 1960s, many electronics firms were selling at multiples of 200 times earnings. The high perceived growth rates did not materialize, causing great declines in stock prices during the early 1970s. In earlier decades, fortunes were made in stocks like IBM and Xerox because the high growth rates were not anticipated by investors. Most recently, we have experienced the dot-com collapse when many Internet stocks were trading at multiples of thousands of times annual earnings. In fact, most Internet stocks had no earnings.

There are two additional factors explaining the PE ratio. The first is the discount rate, R . The previous formula shows that the PE ratio is *negatively* related to the firm's discount

Table 5.1
Selected PE Ratios

Company	Industry	PE Ratio
Ford	Automobiles	7.69
Bear Stearns	Investment banking	11.60
Caterpillar	Heavy equipment	16.79
S&P 500 average	n/a	19.00
Cisco Systems	Computer networking	21.47
Amgen	Biotechnology	27.18
Starbucks	Expensive coffee	50.01

rate. We have already suggested that the discount rate is positively related to the stock's risk or variability. Thus the PE ratio is negatively related to the stock's risk. To see that this is a sensible result, consider two firms, *A* and *B*, behaving as cash cows. The stock market *expects* both firms to have annual earnings of \$1 per share forever. However, the earnings of firm *A* are known with certainty, whereas the earnings of firm *B* are quite variable. A rational stockholder is likely to pay more for a share of firm *A* because of the absence of risk. If a share of firm *A* sells at a higher price and both firms have the same EPS, the PE ratio of firm *A* must be higher.

The second additional factor concerns the firm's choice of accounting methods. Under current accounting rules, companies are given a fair amount of leeway. For example, consider inventory accounting where either FIFO or LIFO may be used. In an inflationary environment, *FIFO* (*first in–first out*) accounting understates the true cost of inventory and hence inflates reported earnings. Inventory is valued according to more recent costs under *LIFO* (*last in–first out*), implying that reported earnings are lower here than they would be under FIFO. Thus LIFO inventory accounting is a more *conservative* method than FIFO. Similar accounting leeway exists for construction costs (*completed contracts* versus *percentage-of-completion methods*) and depreciation (*accelerated depreciation* versus *straight-line depreciation*).

As an example, consider two identical firms, *C* and *D*. Firm *C* uses LIFO and reports earnings of \$2 per share. Firm *D* uses the less conservative accounting assumptions of FIFO and reports earnings of \$3 per share. The market knows that both firms are identical and prices both at \$18 per share. This price–earnings ratio is 9 (= \$18/\$2) for firm *C* and 6 (= \$18/\$3) for firm *D*. Thus, the firm with the more conservative principles has the higher PE ratio.

This last example depends on the assumption that the market sees through differences in accounting treatments. A significant portion of the academic community believes that the market sees through virtually all accounting differences. These academics are adherents of the hypothesis of *efficient capital markets*, a theory that we explore in great detail later in the text. Though many financial people might be more moderate in their beliefs regarding this issue, the consensus view is certainly that many of the accounting differences are seen through. Thus, the proposition that firms with conservative accountants have high PE ratios is widely accepted.

5.9 Stock Market Reporting

If you look through the pages of *The Wall Street Journal* (or another financial newspaper), you will find information about a large number of stocks in several different markets. Figure 5.5 reproduces a small section of the stock page for the New York Stock Exchange from January 20, 2006. Information on most NASDAQ issues is reported in the same way. In Figure 5.5, locate the line for motorcycle maker Harley-Davidson (HarleyDav). With the column headings, the line reads:

52-WEEK		STOCK (DIV)	YLD	PE	VOL 100s	CLOSE	NET CHG
HI	LO		%				
62.49	44.40	HarleyDav .64	1.2	16	70028	54.05	2.56

You can get real-time stock quotes on the Web. See finance.yahoo.com for details.

The first two numbers, 62.49 and 44.40, are the highest and lowest prices for the stock over the past 52 weeks. The .64 is the annual dividend in dollars. Because Harley, like most companies, pays dividends quarterly, this \$.64 is actually the latest quarterly dividend multiplied by 4. So the cash dividend paid was $$.64/4 = $.16$, or 16 cents per share.

Jumping ahead just a bit, "CLOSE" is the closing price of the day (i.e., the last price at which a trade took place before the NYSE closed for the day). The "Net Chg" of 2.56 tells us that the closing price of \$54.05 is \$2.56 higher than it was the day before; so we say that Harley was up 2.56 for the day.

The column marked "Yld %" gives the dividend yield based on the current dividend and the closing price. For Harley, this is $\$.64/54.05 = .0118$, or about 1.2 percent, the number shown. The next column, labeled "PE," is the price-earnings ratio we discussed earlier. It is calculated as the closing price divided by annual earnings per share (based on

Figure 5.5

Sample Stock Quotation from *The Wall Street Journal*

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FRIDAY, JANUARY 20, 2006 CS										
NYSE COMPOSITE TRANSACTIONS										
52-WEEK	YLD	VOL	NET	52-WEEK	YLD	VOL	NET	52-WEEK	YLD	
HI LO STOCK (DIV)	%	100s	CLOSE	HI LO STOCK (DIV)	%	100s	CLOSE	HI LO STOCK (DIV)	%	
F										
33.10 25.15	4.42	1.3	229	32.69	0.48	63.87	43.25	FMC	...	
48.04 28.62	FMC Tech	...	25	8011	47.25	1.03	21	16.18	FNB Gp s 142	5.5
48.11 36.89	FPL Gp s 142	3.3	19	19334	43.24	0.62	13.95	2.00	FTD Grp n	...
30.54 17.01	FTI OnStng	...	26	1264	27.73	0.39	42.30	27.06	FactStRsch s 20	5
48.47 32.26	FairchildCp	...	25	4654	46.37	0.16	3.84	2.11	FairchildCp	...
179.90 126.73	FairfaxFnl 1.40e	9	dd	431	153.70	2.45	43.75	29.90	FairmintHIRT 12	3
18.55 10.91	FairmComm n 1.99	132	dd	1995	12.03	...	31.15	15.42	FalcomBridge 40g	...
35.25 19.40	FamilyDir 38	1.6	19	7450	24.07	-0.11	66.50	41.34	FansiteHse If 1.04	1.9
3.32 1.22	Fedders 09j	1040	1.33	-0.05	23.15	12.89	FedAgrMty A 40	1.9
62.47 15.53	FedAgri C 40	1.4	11	215	29.04	0.24	35.98	46.19	FedRlty 2.22a	3.3
17.95 13.80	FedSgnl 24	1.4	29	3843	16.63	0.50	78.05	54.20	FedDgtStr 1.00	1.4
105.82 26.91	FedExCp 32	3	20	18187	102.72	2.10	18.80	11.78	Felcor 15e	8
14.70 8.63	FHMallMzPrp 91	7.4	...	120	12.25	0.59	22.49	20.13	Ferrogas 2.00	9.3
23.22 16.27	Ferro If 58	3.0	36	1034	19.55	0.38	10.28	5.76	Fiat ADS	...
41.11 27.01	FidNtFnl s 1.00	2.7	6	17489	37.50	0.10	26	19.50	FidNtTitle n 25p	...
45.19 32.70	FnlFad 6.0	1.4	20	1679	42.96	0.41	45.19	32.70	FnlFad 6.0	1.4
10.95 8.23	FstAcptance	...	19	271	10.31	0.14	49.50	30	FstAmCp 72	1.6
29.71 10.37	FstBcp sif 28	2.3	6	1830	12.39	0.16	14.70	12.63	FstCmWthFnl 68f 5.1	15
44.80 34.78	FstHorizNat 1.80	4.7	11	14298	38.62	-0.33	42.80	36.79	FstIndRlty 2.80f 7.2	2.3
73.27 20.89	FstMarhead 24e	7	14	4764	32.64	-0.05	28.30	20.87	FstPotomac 1.20f 4.2	84
40.22 30.56	FstRepBnk s 5.0	1.2	20	2255	40.27	1.16	53.36	47.84	FstFedFnl	...
65.32 38.54	FstEngy 1.80f	3.5	20	7906	51.51	0.40	22.42	12.62	Flagstar 60m	4.2
67.50 53.50	FisherSci	...	26	10349	65.68	0.28	48.49	33.94	FleetWEnt	...
30.61 18.51	FlwrFoods s 4.0	1.5	30	1459	26.92	-0.15	43.35	23.69	Fluorserve If	...
22.42 12.61	FluorCp 64	8	33	6302	81.75	-0.15	79.60	50.47	Fomento ADS 60e	8
29.95 18.74	FrdLocker 36f 1.6	14	6989	22.45	-0.68	13.54	7.57	FrdMotor 4.0	4.9	
45.15 24.77	FrdCdn s 4.77e	8274	39.70	0.75	40.90	29.05	ForestCtyA s 24	6
45.21 32.46	ForestLabs	...	22	19929	43.26	0.63	54.76	31.65	ForestOil	...
90.57 73.02	FortBrdms 1.44	1.8	17	5826	78.59	0.16	44.30	20.65	FoudnCoal 18e	4
23 15.46	4KidsEntn	...	25	681	17.78	0.37	82.20	46.85	FourSeasons 11g	...
31.90 23	FraTelecom 68e	2.9	...	2255	23.15	-0.28	62.49	44.40	HarleyDav 64	1.2
126.69 68.54	HarnanInt 05	...	28	4741	101.80	0.68	15.41	5.96	HrmyGld ADS	...
79.69 57.29	HarrahEntn 1.45	2.0	22	12208	71.83	-0.11	47.60	26.94	Harris s 32	7
31.47 25.04	HarteHanks 20	8	20	1355	26.43	-0.07	89.49	65.35	HrtfndFnl 1.20f 1.4	11
80.30 61.58	HrtfndFnl 1.20f 1.4	11	14241	87.32	0.43	79.17	60.90	HrtfndFnl un 3.00	3.9	
10.48 6.09	Hartmarx	...	13	2425	7.85	0.29	33.69	23.85	HvryFnl n 4.20g 128	...
13.93 7.83	HarvstNatRes	...	7	2983	9.49	0.47	22.35	17.75	Hasbro 36	1.7
18 11.53	HavtryFurn 27f 2.0	1.9	19	938	13.75	0.05	17.69	11.92	HavtryFurn A 25f 1.9	19
29.79 24.60	HawEllnd 1.24	4.7	19	1626	26.50	0.24	4.40	2.45	Head	...
45.75 29.08	Headwaters	...	13	9083	35.98	0.11	28.92	23.13	HlthCrProp 1.68	6.1
39.20 31.15	HlthCr Reit 2.48	6.7	39	1493	37	0.44	20.75	20.75	HlthMgt A 24f 1.1	16
54.11 27.63	HealthNet	...	91	6973	51.21	-0.23	26.34	23.15	HearstArgyl 28	1.2
27.73 20.77	HearthdPymnt n	2032	21.75	0.03	6.22	2.95	MedaMin	...
27.45 18.32	Helco 08f	...	3	29	419	25.39	21.93	14.52	Helco A 08f	4
38.16 33.55	Heinz 1.20	3.5	17	14395	33.99	0.10	11.35	8.36	Hellenic	...
72.58 33.89	Helmpayne 33	5	29	8435	71.66	1.61	34.10	14.52	Herbalife	...
15.55 10	Hercules	...	14	5657	11.10	0.40	37.89	29.16	HeritageProp 2.10	6.1
67.37 52.49	Hershey 98	1.8	27	7545	55.16	0.06	31.43	23.94	Hewlett	...
20.15 13.81	Hexcel	...	dd	7124	19.34	...	12.24	9.78	HghndHsply 56	5.1
31.86 24.27	HghndHsply If 1.70	5.5	67	1698	30.72	0.61	40.45	31.75	HilbRogl 46	1.2
57 44.81	Hillenbrnd 1.13f 2.2	dd	1357	51	0.09	25.81	18.78	Hilton 16	6	
75.40 57.36	Hitachi 96e	1.3	...	591	71.57	1.46	11.95	8.45	Hollngnt A s 20a	2.2
47 32.25	HillyEngyPtnr 2.40	5.8	...	167	41.50	-0.25	43.98	34.56	HomeDpt 60f	1.5
10.14 6.70	Homebank 95e 117	dd	1823	8.12	0.03	30.36	23.75	Hondalnt ADS 30e 1.1	...	
39.50 32.68	Honeywell 83	2.2	20	37918	37.19	0.17	20.80	15.86	HoraceMn 42	2.2
13.10 10.24	HorizonLine n 11p	108	12.25	-0.12	35.44	29.16	HormelFood 56f 1.7	18
42.82 26.83	DR Horton s 40f 1.0	8	31354	38.86	-1.14	38.55	20.10	HrnbkOffshr	...	
45.04 38	HospdyProp 2.92	6.9	26	2593	42.29	0.94	19.81	15.46	Hosr Reit 41e 2.1	60
71.47 45.60	HoustonExplr	...	13	4939	55.95	2.23	73.40	42.50	HovanEnt	...
46.75 34	HSBC Fin un 2.22a	5.2	...	10	42.85	-0.40	32.75	25.94	HuanengPwr 1.21r 4.3	...
49.75 39.19	Hubbell B 1.32	3.2	16	4661	41.35	-1.01	54.20	42.56	Hubbell B 1.32	2.9
42.95 31.31	HudsnUpBp 1.48	3.5	17	1890	42.40	0.11	56.22	43.54	KindrMstrEgy 3.20f 6.3	23
49.25 37.44	KindrMstrEgy 3.20f 6.3	23	4768	50.62	0.52	42.11	24.74	Kindred	...	
74.20 33	KinetiCnpts	...	27	2945	41.20	0.08	18.85	7.50	KingsPharm	...
21.07 14.70	KingswayFin 20g	110	20.74	0.03	10.68	4.61	KitRltyTr 75	4.9
16.52 13.40	KitRltyTr 75	4.9	...	691	15.28	0.26	55.54	37.19	KirbyCp	...
25.07 20.71	KKR Fnl n 72e 3.1	1295	23.25	-0.17	11.30	7.84	KMG America	...
22.51 13.83	KnightTrsp s 16f 8	29	7943	20.89	-0.38	69.09	52.42	KnightRidder 1.48	2.3	
19 14.75	Knoll 25e	1.5	26	356	16.81	-0.07	58.90	43.63	KohlsCp	...
24.10 19	Konami ADS 49e 2.1	8	22.99	0.59	78.65	40.65	Kookmin ADS	...
13.29 9.50	KoorInd	167	10.63	0.08	21.92	13.05	KoreElecPwr 56e 2.6	...
20.40 13.92	KornFerry	19	1458	19.55	34.35	27.44	KrtFoods 92	3.2
20.88 15.15	Kroger	51806	19	-0.06	49.20	27.50	KronosWldwd 1.00	3.3
45.90 24.20	Kubota ADS 37e 9	503	43.20	1.25	82.97	62.58	Kyocera 92e 1.2	...
8.29 1.90	LLE RoyalTr j	324	3.25	-0.01	10.75	5.01	L5I Logic	...
23.92 16.50	LTC Prop x 144f 6.4	15	...	55657	8.86	0.22	84.84	64.66	L3Comm 50	7
11.19 7.75	LaQuinta	7502	11.19	0.01	16.40	10.13	LaZoy Bay 4	2.9
26.50 15.25	LaborDay	3497	21.64	-0.01	58.14	44.63	LabCorp	...
11.87 5.25	Labranche	5746	10.89	0.21	34.31	26.90	LacdedeGrp 1.38	4.5
26.35 19.81	Lafarge ADS 72e 3.1	732	23.57	1.07	70.47	50.85	LafargeHolam 96	1.6
27.56 20.41	LaidlawInt 30e 1.1	12	6321	27.31	0.06	31.69	8.77	LamsonSec	...	
40.40 30.06	LandRise ADS 69e 1.8	591	38.25	0.42	70.09	56.40	LandAmFnl 72	1.1
33 25.39	LandryRes 20	7	13	2704	29.25	1.25	54	43.90	Landuser 1.80f 3.7	...
51.40 29.08	LasVegasSands	66	7548	0.74	38.48	27.34	LasSalleH 1.20	3.1
35.48 20.40	Lazard n 14e 4	3362	34	0.70	15.35	9.20	LeapFrog A	...
54.55 23.28	Leear 1.00	4.1	dd	25619	24.35	1.04	45.96	36.11	LeeEnt 2.1	2.0
131.40 69.82	LeggMason 72	5	34	12325	131.10	4.69	16.15	14.16	LehmHldg 96f 7	13
68.86 50.30	Lennar A 64	1.0	8	13277	62.62	-0.65	46.90	50.30	Lennar B 64	1.1
31.16 18.65	LennoxInt 44f 2.0	1861	30.70	0.42	17.94	8.95	LenorGrp	...
49.65 32.40	Leucadant 25	5	4	2203	49.94	0.46	25.26	20.26	LexgenTrp 1.6f 6.4	4.5
48.15 28.05	LexmarkInt	14	8800	46.75	20.60	18.01	LG PhilipsADS	...
25.03 10.03	Libbey 40	3.8	46	869	10.60	0.34	8.29	1.90	LlE RoyalTr j	...

the most recent four quarters). In the jargon of Wall Street, we might say that Harley “sells for 16 times earnings.”

Finally, the column marked “Vol 100s” tells us how many shares traded during the day (in hundreds). For example, the 70028 for Harley tells us that about 7 million shares changed hands on this day alone. If the average price during the day was \$54 or so, then the dollar volume of transactions was on the order of $\$54 \times 7 \text{ million} = \378 million worth for Harley alone. This was a fairly heavy day of trading in Harley shares, and it serves to illustrate how active the market can be for well-known companies.

If you look over Figure 5.5, you will notice quite a few footnote indicators (small letters) and special symbols. To learn more about these, pick up any *Wall Street Journal* and consult the stock pages.

Summary and Conclusions

In this chapter, we used general present value formulas from the previous chapter to price bonds and stock.

1. Pure discount bonds and perpetuities can be viewed as the polar cases of bonds. The value of a pure discount bond (also called a zero coupon bond, or simply a zero) is:

$$PV = \frac{F}{(1 + R)^T}$$

The value of a perpetuity (also called a *consol*) is:

$$PV = \frac{C}{R}$$

2. Level payment bonds can be viewed as an intermediate case. The coupon payments form an annuity, and the principal repayment is a lump sum. The value of this type of bond is simply the sum of the values of its two parts.
3. The yield to maturity on a bond is the single rate that discounts the payments on the bond to its purchase price.
4. A stock can be valued by discounting its dividends. We mentioned three types of situations:
 - a. The case of zero growth of dividends.
 - b. The case of constant growth of dividends.
 - c. The case of differential growth.
5. An estimate of the growth rate of a stock is needed for the formulas for situations 4(b) or 4(c). A useful estimate of the growth rate is

$$g = \text{Retention ratio} \times \text{Return on retained earnings}$$

6. It is worthwhile to view a share of stock as the sum of its worth if the company behaves like a cash cow (the company does no investing) and the value per share of its growth opportunities. We write the value of a share as:

$$\frac{\text{EPS}}{R} + \text{NPVGO}$$

We showed that, in theory, share price must be the same whether the dividend growth model or the formula here is used.

7. From accounting, we know that earnings are divided into two parts: dividends and retained earnings. Most firms continually retain earnings to create future dividends. One should not

discount earnings to obtain price per share because part of earnings must be reinvested. Only dividends reach the stockholders, and only they should be discounted to obtain share price.

8. We suggested that a firm's price-earnings ratio is a function of three factors:
 - a. The per-share amount of the firm's valuable growth opportunities.
 - b. The risk of the stock.
 - c. The type of accounting method used by the firm.

Concept Questions

1. **Coupon Rate** How does a bond issuer decide on the appropriate coupon rate to set on its bonds? Explain the difference between the coupon rate and the required return on a bond.
2. **Bond Market** What are the implications for bond investors of the lack of transparency in the bond market?
3. **Stock Valuation** Why does the value of a share of stock depend on dividends?
4. **Stock Valuation** A substantial percentage of the companies listed on the NYSE and the NASDAQ don't pay dividends, but investors are nonetheless willing to buy shares in them. How is this possible given your answer to the previous question?
5. **Dividend Policy** Referring to the previous two questions, under what circumstances might a company choose not to pay dividends?
6. **Dividend Growth Model** Under what two assumptions can we use the dividend growth model presented in the chapter to determine the value of a share of stock? Comment on the reasonableness of these assumptions.
7. **Common versus Preferred Stock** Suppose a company has a preferred stock issue and a common stock issue. Both have just paid a \$2 dividend. Which do you think will have a higher price, a share of the preferred or a share of the common?
8. **Growth Rate** In the context of the dividend growth model, is it true that the growth rate in dividends and the growth rate in the price of the stock are identical?
9. **Price-Earnings Ratio** What are the three factors that determine a company's price-earnings ratio?
10. **Stock Valuation** Evaluate the following statement Managers should not focus on the current stock value because doing so will lead to an overemphasis on short-term profits at the expense of long-term profits.

Questions and Problems




BASIC
(Questions 1-9)

1. **Valuing Bonds** What is the price of a 10-year, pure discount bond paying \$1,000 at maturity if the YTM is
 - a. 5 percent?
 - b. 10 percent?
 - c. 15 percent?
2. **Valuing Bonds** Microhard has issued a bond with the following characteristics:
 - Par: \$1,000
 - Time to maturity: 20 years
 - Coupon rate: 8 percent
 - Semiannual payments
 Calculate the price of this bond if the YTM is
 - a. 8 percent.
 - b. 10 percent.
 - c. 6 percent.
3. **Bond Yields** Raines Umbrella Corp. issued 12-year bonds 2 years ago at a coupon rate of 8.6 percent. The bonds make semiannual payments. If these bonds currently sell for 97 percent of par value, what is the YTM?

4. **Stock Values** The Brennan Co. just paid a dividend of \$1.40 per share on its stock. The dividends are expected to grow at a constant rate of 6 percent per year indefinitely. If investors require a 12 percent return on the Brennan Co. stock, what is the current price? What will the price be in three years? In 15 years?
5. **Stock Values** The next dividend payment by MUG, Inc., will be \$3.10 per share. The dividends are anticipated to maintain a 5 percent growth rate forever. If MUG stock currently sells for \$48.00 per share, what is the required return?
6. **Stock Values** Warren Corporation will pay a \$3.60 per share dividend next year. The company pledges to increase its dividend by 4.5 percent per year indefinitely. If you require a 13 percent return on your investment, how much will you pay for the company's stock today?
7. **Stock Valuation** Suppose you know that a company's stock currently sells for \$70 per share and the required return on the stock is 12 percent. You also know that the total return on the stock is evenly divided between a capital gains yield and a dividend yield. If it's the company's policy to always maintain a constant growth rate in its dividends, what is the current dividend per share?
8. **Stock Valuation** Gruber Corp. pays a constant \$12 dividend on its stock. The company will maintain this dividend for the next eight years and will then cease paying dividends forever. If the required return on this stock is 10 percent, what is the current share price?
9. **Growth Rate** The newspaper reported last week that Bradley Enterprises earned \$20 million this year. The report also stated that the firm's return on equity is 14 percent. Bradley retains 60 percent of its earnings. What is the firm's earnings growth rate? What will next year's earnings be?
10. **Bond Price Movements** Miller Corporation has a premium bond making semiannual payments. The bond pays an 8 percent coupon, has a YTM of 6 percent, and has 13 years to maturity. The Modigliani Company has a discount bond making semiannual payments. This bond pays a 6 percent coupon, has a YTM of 8 percent, and also has 13 years to maturity. If interest rates remain unchanged, what do you expect the price of these bonds to be 1 year from now? In 3 years? In 8 years? In 12 years? In 13 years? What's going on here? Illustrate your answers by graphing bond prices versus time to maturity.
11. **Bond Yields** Stealers Wheel Software has 8.4 percent coupon bonds on the market with nine years to maturity. The bonds make semiannual payments and currently sell for 104 percent of par. What is the current yield on the bonds? The YTM? The effective annual yield?
12. **Bond Yields** Petty Co. wants to issue new 20-year bonds for some much-needed expansion projects. The company currently has 8 percent coupon bonds on the market that sell for \$1,095, make semiannual payments, and mature in 20 years. What coupon rate should the company set on its new bonds if it wants them to sell at par?
13. **Stock Valuation** Ferson, Inc., just paid a dividend of \$3.00 on its stock. The growth rate in dividends is expected to be a constant 5 percent per year indefinitely. Investors require a 16 percent return on the stock for the first three years, a 14 percent return for the next three years, and then an 11 percent return thereafter. What is the current share price for Ferson stock?
14. **Nonconstant Growth** Metallica Bearings, Inc., is a young start-up company. No dividends will be paid on the stock over the next nine years because the firm needs to plow back its earnings to fuel growth. The company will pay an \$8 per share dividend in 10 years and will increase the dividend by 6 percent per year thereafter. If the required return on this stock is 13 percent, what is the current share price?
15. **Nonconstant Dividends** Corn, Inc., has an odd dividend policy. The company has just paid a dividend of \$9 per share and has announced that it will increase the dividend by \$3 per share for each of the next four years, and then never pay another dividend. If you require an 11 percent return on the company's stock, how much will you pay for a share today?
16. **Nonconstant Dividends** South Side Corporation is expected to pay the following dividends over the next four years: \$8, \$6, \$3, and \$2. Afterward the company pledges to maintain a constant 5 percent growth rate in dividends forever. If the required return on the stock is 13 percent, what is the current share price?

INTERMEDIATE
(Questions 10–31)



17. **Nonconstant Growth** Rizzi Co. is growing quickly. Dividends are expected to grow at a 25 percent rate for the next three years, with the growth rate falling off to a constant 7 percent thereafter. If the required return is 13 percent and the company just paid a \$2.80 dividend, what is the current share price?
18. **Nonconstant Growth** Janicek Corp. is experiencing rapid growth. Dividends are expected to grow at 30 percent per year during the next three years, 18 percent over the following year, and then 8 percent per year indefinitely. The required return on this stock is 14 percent, and the stock currently sells for \$70.00 per share. What is the projected dividend for the coming year?
19. **Finding the Dividend** Hollin Corporation stock currently sells for \$50 per share. The market requires a 14 percent return on the firm's stock. If the company maintains a constant 8 percent growth rate in dividends, what was the most recent dividend per share paid on the stock?
-  20. **Valuing Preferred Stock** Mark Bank just issued some new preferred stock. The issue will pay a \$9 annual dividend in perpetuity, beginning six years from now. If the market requires a 7 percent return on this investment, how much does a share of preferred stock cost today?
21. **Negative Growth** Calamity Mining Company's iron ore reserves are being depleted, and its costs of recovering a declining quantity of ore are rising each year. As a result, the company's earnings are declining at a rate of 10 percent per year. If the dividend per share to be paid tomorrow is \$5 and the required rate of return is 14 percent, what is the value of the firm's stock? Assume that the dividend payments are based on a fixed percentage of the firm's earnings.
22. **Nonconstant Growth and Quarterly Dividends** Pasqually Mineral Water, Inc., will pay a quarterly dividend per share of \$1 at the end of each of the next 12 quarters. Thereafter the dividend will grow at a quarterly rate of 0.5 percent forever. The appropriate rate of return on the stock is 10 percent, compounded quarterly. What is the current stock price?
-  23. **Nonconstant Growth** To buy back its own shares, Pennzoil Co. has decided to suspend its dividends for the next two years. It will resume its annual cash dividend of \$2.00 in year 3 and year 4. Thereafter its dividend payments will grow at an annual growth rate of 6 percent forever. The required rate of return on Pennzoil's stock is 16 percent. According to the discounted dividend model, what should Pennzoil's current share price be?
24. **Finding the Dividend** Allen, Inc., is expected to pay equal dividends at the end of each of the next two years. Thereafter, the dividend will grow at a constant annual rate of 4 percent forever. The current stock price is \$30. What is next year's dividend payment if the required rate of return is 12 percent?
25. **Finding the Required Return** Juggernaut Satellite Corporation earned \$10 million for the fiscal year ending yesterday. The firm also paid out 25 percent of its earnings as dividends yesterday. The firm will continue to pay out 25 percent of its earnings as annual, end-of-year dividends. The remaining 75 percent of earnings is retained by the company for use in projects. The company has 1.25 million shares of common stock outstanding. The current stock price is \$40. The historical return on equity (ROE) of 11 percent is expected to continue in the future. What is the required rate of return on the stock?
-  26. **Dividend Growth** Four years ago, Bling Diamond, Inc., paid a dividend of \$.90 per share. Bling paid a dividend of \$1.66 per share yesterday. Dividends will grow over the next five years at the same rate they grew over the last four years. Thereafter dividends will grow at 8 percent per year. The required return on the stock is 18 percent. What will Bling Diamond's cash dividend be in seven years?
27. **Price–Earnings Ratio** Consider Pacific Energy Company and U.S. Bluechips, Inc., both of which reported earnings of \$800,000. Without new projects, both firms will continue to generate earnings of \$800,000 in perpetuity. Assume that all earnings are paid as dividends and that both firms require a 15 percent rate of return.
- What is the current PE ratio for each company?
 - Pacific Energy Company has a new project that will generate additional earnings of \$100,000 each year in perpetuity. Calculate the new PE ratio of the company.

- c. U. S. Bluechips has a new project that will increase by \$200,000 in perpetuity. Calculate the new PE ratio of the firm.
28. **Growth Opportunities** The Stambaugh Corporation currently has earnings per share of \$7.00. The company has no growth and pays out all earnings as dividends. It has a new project that will require an investment of \$1.75 per share in one year. The project will only last two years and will increase earnings in the two years following the investment by \$1.90 and \$2.10, respectively. Investors require a 12 percent return on Stambaugh stock.
- What is the value per share of the company's stock assuming the firm does not undertake the investment opportunity?
 - If the company does undertake the investment, what is the value per share now?
 - Again assume the company undertakes the investment. What will the price per share be four years from today?
29. **Growth Opportunities** Rite Bite Enterprises sells toothpicks. Gross revenues last year were \$3 million, and total costs were \$1.5 million. Rite Bite has 1 million shares of common stock outstanding. Gross revenues and costs are expected to grow at 5 percent per year. Rite Bite pays no income taxes. All earnings are paid out as dividends.
- If the appropriate discount rate is 15 percent and all cash flows are received at year's end, what is the price per share of Rite Bite stock?
 - Rite Bite has decided to produce toothbrushes. The project requires an immediate outlay of \$15 million. In one year, another outlay of \$5 million will be needed. The year after that, earnings will increase by \$6 million. That profit level will be maintained in perpetuity. What effect will undertaking this project have on the price per share of the stock?
30. **Growth Opportunities** California Real Estate, Inc., expects to earn \$110 million per year in perpetuity if it does not undertake any new projects. The firm has an opportunity to invest \$12 million today and \$7 million in one year in real estate. The new investment will generate annual earnings of \$10 million in perpetuity, beginning two years from today. The firm has 20 million shares of common stock outstanding, and the required rate of return on the stock is 15 percent. Land investments are not depreciable. Ignore taxes.
- What is the price of a share of stock if the firm does not undertake the new investment?
 - What is the value of the investment?
 - What is the per-share stock price if the firm undertakes the investment?
31. **Growth Opportunities** The annual earnings of Avalanche Skis, Inc., will be \$5 per share in perpetuity if the firm makes no new investments. Under such a situation the firm would pay out all of its earnings as dividends. Assume the first dividend will be received exactly one year from now.
- Alternatively, assume that three years from now, and in every subsequent year in perpetuity, the company can invest 25 percent of its earnings in new projects. Each project will earn 40 percent at year-end in perpetuity. The firm's discount rate is 14 percent.
- What is the price per share of Avalanche Skis, Inc., stock today without the company making the new investment?
 - If Avalanche announces that the new investment will be made, what will the per-share stock price be today?
32. **Components of Bond Returns** Bond P is a premium bond with a 10 percent coupon. Bond D is a 6 percent coupon bond currently selling at a discount. Both bonds make annual payments, have a YTM of 8 percent, and have five years to maturity. What is the current yield for Bond P? For Bond D? If interest rates remain unchanged, what is the expected capital gains yield over the next year for Bond P? For Bond D? Explain your answers and the interrelationship among the various types of yields.
33. **Holding Period Yield** The YTM on a bond is the interest rate you earn on your investment if interest rates don't change. If you actually sell the bond before it matures, your realized return is known as the holding period yield (HPY).

CHALLENGE
(Questions 32–40)

- a. Suppose that today you buy an 8 percent annual coupon bond for \$1,150. The bond has 10 years to maturity. What rate of return do you expect to earn on your investment?
- b. Two years from now, the YTM on your bond has declined by 1 percent, and you decide to sell. What price will your bond sell for? What is the HPY on your investment? Compare this yield to the YTM when you first bought the bond. Why are they different?
34. **Valuing Bonds** The Mallory Corporation has two different bonds currently outstanding. Bond M has a face value of \$20,000 and matures in 20 years. The bond makes no payments for the first six years, then pays \$1,200 every six months over the subsequent eight years, and finally pays \$1,500 every six months over the last six years. Bond N also has a face value of \$20,000 and a maturity of 20 years; it makes no coupon payments over the life of the bond. If the required return on both these bonds is 10 percent compounded semiannually, what is the current price of Bond M? Of Bond N?
35. **Capital Gains versus Income** Consider four different stocks, all of which have a required return of 15 percent and a most recent dividend of \$4.50 per share. Stocks W, X, and Y are expected to maintain constant rates in dividends for the foreseeable future of 10 percent, 0 percent, and -5 percent per year, respectively. Stock Z is a growth stock that will increase its dividend by 20 percent for the next two years and then maintain a constant 12 percent growth rate thereafter. What is the dividend yield for each of these four stocks? What is the expected capital gains yield? Discuss the relationship among the various returns that you find for each of these stocks.
36. **Stock Valuation** Most corporations pay quarterly rather than annual dividends on their common stock. Barring any unusual circumstances during the year, the board raises, lowers, or maintains the current dividend once a year and then pays this dividend out in equal quarterly installments to its shareholders.
- a. Suppose a company currently pays a \$3.00 annual dividend on its common stock in a single annual installment, and management plans on raising this dividend by 6 percent per year indefinitely. If the required return on this stock is 14 percent, what is the current share price?
- b. Now suppose that the company in (a) actually pays its annual dividend in equal quarterly installments; thus this company has just paid a \$.75 dividend per share, as it has for the previous three quarters. What is your value for the current share price now? (*Hint:* Find the equivalent annual end-of-year dividend for each year.) Comment on whether you think that this model of stock valuation is appropriate.
37. **Growth Opportunities** Lewin Skis, Inc., (today) expects to earn \$6 per share for each of the future operating periods (beginning at time 1) if the firm makes no new investments and returns the earnings as dividends to the shareholders. However, Clint Williams, president and CEO, has discovered an opportunity to retain and invest 30 percent of the earnings beginning three years from today. This opportunity to invest will continue for each period indefinitely. He expects to earn 12 percent on this new equity investment, the return beginning one year after each investment is made. The firm's equity discount rate is 14 percent throughout.
- a. What is the price per share of Lewin Skis, Inc., stock without making the new investment?
- b. If the new investment is expected to be made, per the preceding information, what would the price of the stock be now?
- c. Suppose the company could increase the investment in the project by whatever amount it chose. What would the retention ratio need to be to make this project attractive?
38. **Nonconstant Growth** Storico Co. just paid a dividend of \$3.50 per share. The company will increase its dividend by 20 percent next year and will then reduce its dividend growth rate by 5 percentage points per year until it reaches the industry average of 5 percent dividend growth, after which the company will keep a constant growth rate forever. If the required return on Storico stock is 13 percent, what will a share of stock sell for today?
39. **Nonconstant Growth** This one's a little harder. Suppose the current share price for the firm in the previous problem is \$98.65 and all the dividend information remains the same. What required return must investors be demanding on Storico stock? (*Hint:* Set up the valuation formula with all the relevant cash flows, and use trial and error to find the unknown rate of return.)

40. **Growth Opportunities** Shane, Inc., has earnings of \$10 million and is projected to grow at a constant rate of 5 percent forever because of the benefits gained from the learning curve. Currently all earnings are paid out as dividends. The company plans to launch a new project two years from now that would be completely internally funded and require 20 percent of the earnings that year. The project would start generating revenues one year after the launch of the project, and the earnings from the new project in any year are estimated to be constant at \$5 million. The company has 10 million shares of stock outstanding. Estimate the value of Shane stock. The discount rate is 10 percent.

S&P Problems

STANDARD
& POOR'S

www.mhhe.com/edumarketinsight

- Dividend Discount Model** Enter the ticker symbol “WMT” for Wal-Mart. Using the most recent balance sheet and income statement under the “Excel Analytics” link, calculate the sustainable growth rate for Wal-Mart. Now download the “Mthly. Adj. Price” and find the closing stock price for the same month as the balance sheet and income statement you used. What is the implied required return on Wal-Mart according to the dividend growth model? Does this number make sense? Why or why not?
- Growth Opportunities** Assume that investors require an 11 percent return on Harley-Davidson (HDI) stock. Under the “Excel Analytics” link find the “Mthly. Adj. Price” and find the closing price for the month of the most recent fiscal year end for HDI. Using this stock price and the EPS for the most recent year, calculate the NPVGO for Harley-Davidson. What is the appropriate PE ratio for Harley-Davidson using these calculations?

Stock Valuation at Ragan Thermal Systems

Ragan Thermal Systems, Inc., was founded nine years ago by brother and sister Carrington and Genevieve Ragan. The company manufactures and installs commercial heating, ventilation, and cooling (HVAC) units. Ragan has experienced rapid growth because of a proprietary technology that increases the energy efficiency of its systems. The company is equally owned by Carrington and Genevieve. The original agreement between the siblings gave each 50,000 shares of stock. In the event either wished to sell the stock, the shares first had to be offered to the other at a discounted price.

Although neither sibling wants to sell any shares at this time, they have decided they should value their holdings in the company for financial planning purposes. To accomplish this, they have gathered the following information about their main competitors.

Ragan Thermal Systems, Inc., Competitors					
	EPS	DPS	Stock Price	ROE	R
Arctic Cooling, Inc.	\$.82	\$.16	\$15.19	11%	10%
National Heating & Cooling	1.32	.52	12.49	14	13
Expert HVAC Corp.	-.47	.54	48.60	14	12
Industry average	\$0.56	\$0.41	\$25.43	13%	11.67%

Expert HVAC Corp.’s negative earnings per share (EPS) were the result of an accounting write-off last year. Without the write-off, EPS for the company would have been \$2.34.

Last year, Ragan had an EPS of \$4.32 and paid a dividend to Carrington and Genevieve of \$54,000 each. The company also had a return on equity of 25 percent. The siblings believe a required return for the company of 20 percent is appropriate.

- Assuming the company continues its current growth rate, what is the value per share of the company’s stock?

2. To verify their calculations, Carrington and Genevieve have hired Josh Schlessman as a consultant. Josh was previously an equity analyst, and he has covered the HVAC industry. Josh has examined the company's financial statements as well as those of its competitors. Although Ragan currently has a technological advantage, Josh's research indicates that Ragan's competitors are investigating other methods to improve efficiency. Given this, Josh believes that Ragan's technological advantage will last for only the next five years. After that period, the company's growth will likely slow to the industry average. Additionally, Josh believes that the required return the company uses is too high. He believes the industry average required return is more appropriate. Under Josh's assumptions, what is the estimated stock price?
3. What is the industry average price–earnings ratio? What is Ragan's price–earnings ratio? Comment on any differences and explain why they may exist.
4. Assume the company's growth rate declines to the industry average after five years. What percentage of the stock's value is attributable to growth opportunities?
5. Assume the company's growth rate slows to the industry average in five years. What future return on equity does this imply?
6. After discussions with Josh, Carrington and Genevieve agree that they would like to try to increase the value of the company stock. Like many small business owners, they want to retain control of the company and do not want to sell stock to outside investors. They also feel that the company's debt is at a manageable level and do not want to borrow more money. What steps can they take to increase the price of the stock? Are there any conditions under which this strategy would not increase the stock price?

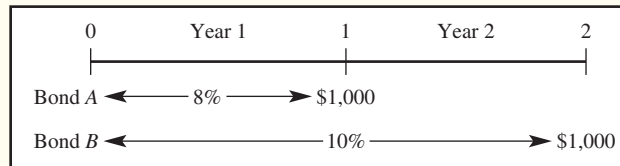
Appendix 5A The Term Structure of Interest Rates, Spot Rates, and Yield to Maturity

To access the appendix for this chapter, please go to www.mhhe.com/rwj.

Appendix 5A The Term Structure of Interest Rates, Spot Rates, and Yield to Maturity

In the main body of this chapter, we have assumed that the interest rate is constant over all future periods. In reality, interest rates vary through time. This occurs primarily because inflation rates are expected to differ through time.

To illustrate, we consider two zero coupon bonds. Bond *A* is a one-year bond and bond *B* is a two-year bond. Both have face values of \$1,000. The one-year interest rate, r_1 , is 8 percent. The two-year interest rate, r_2 , is 10 percent. These two rates of interest are examples of *spot rates*. Perhaps this inequality in interest rates occurs because inflation is expected to be higher over the second year than over the first year. The two bonds are depicted in the following time chart:



We can easily calculate the present value for bond *A* and bond *B* as follows:

$$PV_A = \$925.93 = \frac{\$1,000}{1.08}$$

$$PV_B = \$826.45 = \frac{\$1,000}{(1.10)^2}$$

Of course, if PV_A and PV_B were observable and the spot rates were not, we could determine the spot rates using the PV formula, because:

$$PV_A = \$925.93 = \frac{\$1,000}{(1 + r_1)} \rightarrow r_1 = 8\%$$

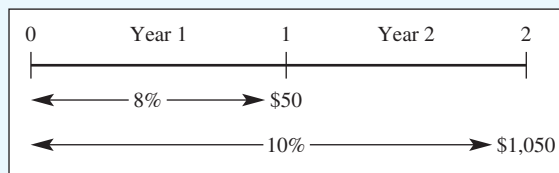
and:

$$PV_B = \$826.45 = \frac{\$1,000}{(1 + r_2)^2} \rightarrow r_2 = 10\%$$

Now we can see how the prices of more complicated bonds are determined. Try to do the next example. It illustrates the difference between spot rates and yields to maturity.

EXAMPLE 5A.1

On the Spot Given the spot rates r_1 equals 8 percent and r_2 equals 10 percent, what should a 5 percent coupon, two-year bond cost? The cash flows C_1 and C_2 are illustrated in the following time chart:



The bond can be viewed as a portfolio of zero coupon bonds with one- and two-year maturities. Therefore:

$$PV = \frac{\$50}{1 + 0.08} + \frac{\$1,050}{(1 + 0.10)^2} = \$914.06 \quad (\text{A.1})$$

(continued)

We now want to calculate a single rate for the bond. We do this by solving for y in the following equation:

$$\$914.06 = \frac{\$50}{1+y} + \frac{\$1,050}{(1+y)^2} \quad (\text{A.2})$$

In Equation A.2, y equals 9.95 percent. As mentioned in the chapter, we call y the *yield to maturity* on the bond. Solving for y for a multiyear bond is generally done by means of trial and error.¹ Although this can take much time with paper and pencil, it is virtually instantaneous on a handheld calculator.

It is worthwhile to contrast Equations A.1 and A.2. In A.1, we use the marketwide spot rates to determine the price of the bond. Once we get the bond price, we use A.2 to calculate its yield to maturity. Because Equation A.1 employs two spot rates whereas only one appears in A.2, we can think of yield to maturity as some sort of average of the two spot rates.²

Using these spot rates, the yield to maturity of a two-year coupon bond whose coupon rate is 12 percent and PV equals \$1,036.73 can be determined by:

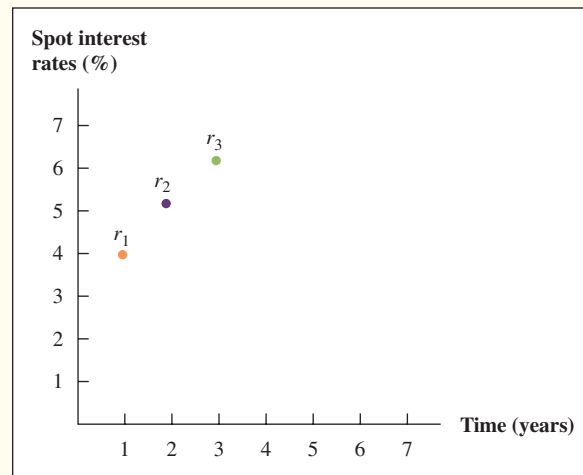
$$\$1,036.73 = \frac{\$120}{1+r} + \frac{\$1,120}{(1+r)^2} \rightarrow r = 9.89\%$$

As these calculations show, two bonds with the same maturity will usually have different yields to maturity if the coupons differ.

Graphing the Term Structure The *term structure* describes the relationship of spot rates with different maturities. Figure 5A.1 graphs a particular term structure. In Figure 5A.1 the spot rates are increasing with longer maturities—that is, $r_3 > r_2 > r_1$. Graphing the term structure is easy if we can observe spot rates. Unfortunately this can be done only if there are enough zero coupon government bonds.

A given term structure, such as that in Figure 5A.1, exists for only a moment in time—say 10:00 a.m., July 30, 2006. Interest rates are likely to change in the next minute, so that a different (though quite similar) term structure would exist at 10:01 a.m.

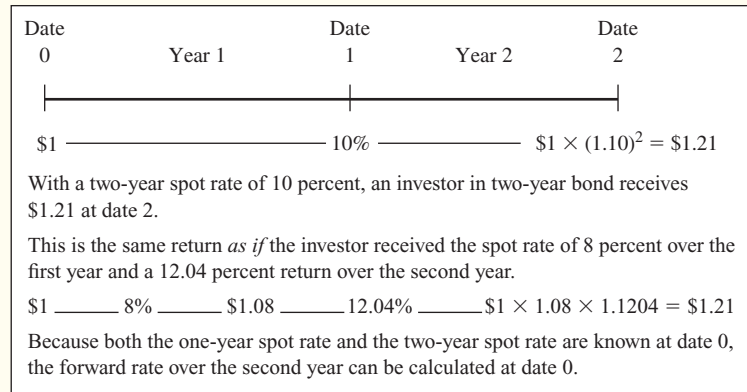
Figure 5A.1
The Term Structure of
Interest Rates



¹The quadratic formula may be used to solve for y for a two-year bond. However, formulas generally do not apply for bonds with more than four payment dates.

²Yield to maturity is not a simple average of r_1 and r_2 . Rather, financial economists speak of it as a time-weighted average of r_1 and r_2 .

Figure 5A.2
Breakdown of a Two-Year Spot Rate into a One-Year Spot Rate and Forward Rate over the Second Year



Explanations of the Term Structure

Figure 5A.1 showed one of many possible relationships between the spot rate and maturity. We now want to explore the relationship in more detail. We begin by defining a new term, the forward rate. Next, we relate this forward rate to future interest rates. Finally we consider alternative theories of the term structure.

Definition of Forward Rate Earlier in this appendix, we developed a two-year example where the spot rate over the first year is 8 percent and the spot rate over the two years is 10 percent. Here, an individual investing \$1 in a two-year zero coupon bond would have $\$1 \times (1.10)^2$ in two years.

To pursue our discussion, it is worthwhile to rewrite:³

$$\$1 \times (1.10)^2 = \$1 \times 1.08 \times 1.1204 \quad (\text{A.3})$$

Equation A.3 tells us something important about the relationship between one- and two-year rates. When an individual invests in a two-year zero coupon bond yielding 10 percent, his wealth at the end of two years is the same as if he received an 8 percent return over the first year and a 12.04 percent return over the second year. This hypothetical rate over the second year, 12.04 percent, is called the *forward rate*. Thus, we can think of an investor with a two-year zero coupon bond as getting the one-year spot rate of 8 percent and *locking in* 12.04 percent over the second year. This relationship is presented in Figure 5A.2.

More generally, if we are given spot rates r_1 and r_2 , we can always determine the forward rate, f_2 , such that:

$$(1 + r_2)^2 = (1 + r_1) \times (1 + f_2) \quad (\text{A.4})$$

We solve for f_2 , yielding:

$$f_2 = \frac{(1 + r_2)^2}{1 + r_1} - 1 \quad (\text{A.5})$$

³12.04 percent is equal to:

$$\frac{(1.10)^2}{(1.08)} - 1$$

when rounding is performed after four digits.

EXAMPLE 5A.2

Looking Forward If the one-year spot rate is 7 percent and the two-year spot rate is 12 percent, what is f_2 ?

We plug in Equation A.5, yielding:

$$f_2 = \frac{(1.12)^2}{1.07} - 1 = 17.23\%$$

Consider an individual investing in a two-year zero coupon bond yielding 12 percent. We say it is as if he receives 7 percent over the first year and simultaneously locks in 17.23 percent over the second year. Note that both the one-year spot rate and the two-year spot rate are known at date 0. Because the forward rate is calculated from the one-year and two-year spot rates, it can be calculated at date 0 as well.

Forward rates can be calculated over later years as well. The general formula is:

$$f_n = \frac{(1 + r_n)^n}{(1 + r_{n-1})^{n-1}} - 1 \quad (\text{A.6})$$

where f_n is the forward rate over the n th year, r_n is the n -year spot rate, and r_{n-1} is the spot rate for $n - 1$ years.

EXAMPLE 5A.3

Forward Rates Assume the following set of rates:

Year	Spot Rate
1	5%
2	6
3	7
4	6

What are the forward rates over each of the four years?

The forward rate over the first year is, *by definition*, equal to the one-year spot rate. Thus, we do not generally speak of the forward rate over the first year. The forward rates over the later years are:

$$f_2 = \frac{(1.06)^2}{1.05} - 1 = 7.01\%$$

$$f_3 = \frac{(1.07)^3}{(1.06)^2} - 1 = 9.03\%$$

$$f_4 = \frac{(1.06)^4}{(1.07)^3} - 1 = 3.06\%$$

An individual investing \$1 in the two-year zero coupon bond receives \$1.1236 [$\$1 \times (1.06)^2$] at date 2. He can be viewed as receiving the one-year spot rate of 5 percent over the first year and receiving the forward rate of 7.01 percent over the second year. An individual investing \$1 in a three-year zero coupon bond receives \$1.2250 [$\$1 \times (1.07)^3$] at date 3. She can be viewed as receiving the two-year spot rate of 6 percent over the first two years and receiving the forward rate of 9.03 percent over the third year.

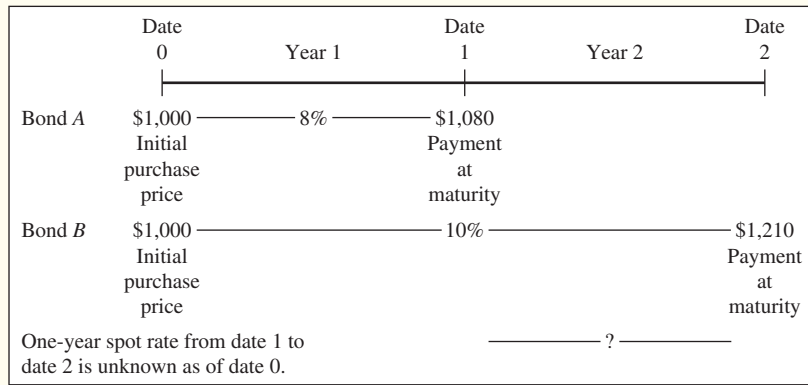
An individual investing \$1 in a four-year zero coupon bond receives \$1.2625 [$\$1 \times (1.06)^4$] at date 4. He can be viewed as receiving the three-year spot rate of 7 percent over the first three years and receiving the forward rate of 3.06 percent over the fourth year.

Note that all of the four spot rates in this problem are known at date 0. Because the forward rates are calculated from the spot rates, they can be determined at date 0 as well.

The material in this appendix is likely to be difficult for a student exposed to term structure for the first time. It helps to state what the student should know at this point. Given Equations A.5 and A.6, a student should be able to calculate a set of forward rates given a set of spot rates. This can simply be viewed as a mechanical computation. In addition to the calculations, a student should understand the intuition of Figure 5A.2.

We now turn to the relationship between the forward rate and the expected spot rates in the future.

Estimating the Price of a Bond at a Future Date In the example from the body of this chapter, we considered zero coupon bonds paying \$1,000 at maturity and selling at a discount prior to maturity. We now wish to change the example slightly. Now each bond initially sells at par so that payment at maturity is above \$1,000.⁴ Keeping the spot rates at 8 percent and 10 percent, we have the following:



The payments at maturity are \$1,080 and \$1,210 for the one- and two-year zero coupon bonds, respectively. The initial purchase price of \$1,000 for each bond is determined as follows:

$$\begin{aligned}
 \$1,000 &= \frac{\$1,080}{1.08} \\
 \$1,000 &= \frac{\$1,210}{(1.10)^2}
 \end{aligned}$$

We refer to the one-year bond as bond *A* and the two-year bond as bond *B*.

There will be a different one-year spot rate when date 1 arrives. This will be the spot rate from date 1 to date 2. We can also call it the spot rate over year 2. This spot rate is not known as of date 0. For example, should the rate of inflation rise between date 0 and date 1, the spot rate over year 2 would likely be high. Should the rate of inflation fall between date 0 and date 1, the spot rate over year 2 would likely be low.

Now that we have determined the price of each bond at date 0, we want to determine what the price of each bond will be at date 1. The price of the one-year bond (bond *A*) must be \$1,080 at date 1 because the payment at maturity is made then. The hard part is determining what the price of the two-year bond (bond *B*) will be at that time.

Suppose we find that, on date 1, the one-year spot rate from date 1 to date 2 is 6 percent. We state that this is the one-year spot rate *over* year 2. This means that you can invest \$1,000 at date 1 and receive \$1,060 (\$1,000 × 1.06) at date 2. Because one year has already passed for bond *B*, the bond has only one year left. Because bond *B* pays \$1,210 at date 2, its value at date 1 is:

$$\$1,141.51 = \frac{\$1,210}{1.06} \tag{A.7}$$

⁴This change in assumptions simplifies our presentation but does not alter any of our conclusions.

Table 5A.1

Price of Bond *B* at Date 1 as a Function of Spot Rate over Year 2

Price of Bond <i>B</i> at Date 1	Spot Rate over Year 2
$\$1,141.51 = \frac{\$1,210}{1.06}$	6%
$\$1,130.84 = \frac{\$1,210}{1.07}$	7%
$\$1,061.40 = \frac{\$1,210}{1.14}$	14%

Note that no one *knew* ahead of time the price that bond *B* would sell for on date 1 because no one knew that the one-year spot rate over year 2 would be 6 percent.

Suppose the one-year spot rate beginning at date 1 turned out not to be 6 percent, but to be 7 percent instead. This means that you can invest \$1,000 at date 1 and receive \$1,070 ($\$1,000 \times 1.07$) at date 2. In this case, the value of bond *B* at date 1 would be:

$$\$1,130.84 = \frac{\$1,210}{1.07} \quad (\text{A.8})$$

Finally, suppose that the one-year spot rate at date 1 turned out to be neither 6 percent nor 7 percent, but 14 percent instead. This means that you can invest \$1,000 at date 1 and receive \$1,140 ($\$1,000 \times 1.14$) at date 2. In this case, the value of bond *B* at date 1 would be:

$$\$1,061.40 = \frac{\$1,210}{1.14}$$

These possible bond prices are represented in Table 5A.1. The price that bond *B* will sell for on date 1 is not known before date 1 because the one-year spot rate prevailing over year 2 is not known until date 1.

It is important to reemphasize that although the forward rate is known at date 0, the one-year spot rate beginning at date 1 is *unknown* ahead of time. Thus, the price of bond *B* at date 1 is unknown ahead of time. Prior to date 1, we can speak only of the amount that bond *B* is *expected* to sell for on date 1. We write this as follows:⁵

The Amount That Bond *B* Is Expected to Sell for on Date 1:

$$\frac{\$1,210}{1 + \text{Spot rate expected over year 2}} \quad (\text{A.9})$$

It is worthwhile making two points now. First, because each individual is different, the expected value of bond *B* differs across individuals. Later we will speak of a consensus expected value across investors. Second, Equation A.9 represents one's forecast of the price that the bond will be selling for on date 1. The forecast is made ahead of time—that is, on date 0.

The Relationship between Forward Rate over Second Year and Spot Rate Expected over Second Year

Given a forecast of bond *B*'s price, an investor can choose one of two strategies at date 0:

1. Buy a one-year bond. Proceeds at date 1 would be:

$$\$1,080 = \$1,000 \times 1.08 \quad (\text{A.10})$$

⁵Technically, Equation A.9 is only an approximation due to *Jensen's inequality*. That is, expected values are:

$$\frac{\$1,210}{1 + \text{Spot rate}} > \frac{\$1,210}{1 + \text{Spot rate expected over year 2}}$$

However, we ignore this very minor issue in the rest of the analysis.

2. Buy a two-year bond but sell at date 1. *Expected* proceeds would be:

$$\frac{\$1,000 \times (1.10)^2}{1 + \text{Spot rate expected over year 2}} \quad (\text{A.11})$$

Given our discussion of forward rates, we can rewrite Equation A.11 as:

$$\frac{\$1,000 \times 1.08 \times 1.1204}{1 + \text{Spot rate expected over year 2}} \quad (\text{A.12})$$

(Remember that 12.04 percent was the forward rate over year 2; that is, $f_2 = 12.04\%$.)

Under what condition will the return from strategy 1 equal the expected return from strategy 2? In other words, under what condition will Equation A.10 equal Equation A.12?

The two strategies will yield the same expected return only when:

$$12.04\% = \text{Spot rate expected over year 2} \quad (\text{A.13})$$

In other words, if the forward rate equals the expected spot rate, one would expect to earn the same return over the first year whether one

- Invested in a one-year bond.
- Invested in a two-year bond but sold after one year.

The Expectations Hypothesis

Equation A.13 seems fairly reasonable. That is, it is reasonable that investors would set interest rates in such a way that the forward rate would equal the spot rate expected by the marketplace a year from now.⁶ For example, imagine that individuals in the marketplace do not concern themselves with risk. If the forward rate, f_2 , is less than the spot rate expected over year 2, individuals desiring to invest for one year would always buy a one-year bond. That is, our work shows that an individual investing in a two-year bond but planning to sell at the end of one year would expect to earn less than if he simply bought a one-year bond.

Equation A.13 was stated for the specific case where the forward rate was 12.04 percent. We can generalize this as follows:

Expectations Hypothesis:

$$f_2 = \text{Spot rate expected over year 2} \quad (\text{A.14})$$

Equation A.14 says that the forward rate over the second year is set to the spot rate that people expect to prevail over the second year. This is called the *expectations hypothesis*. It states that investors will set interest rates such that the forward rate over the second year is equal to the one-year spot rate expected over the second year.

Liquidity Preference Hypothesis

At this point, many students think that Equation A.14 *must* hold. However, note that we developed Equation A.14 by assuming that investors were risk-neutral. Suppose, alternatively, that investors are averse to risk.

Which strategy would appear more risky for an individual who wants to invest for one year?

1. Invest in a one-year bond.
2. Invest in a two-year bond but sell at the end of one year.

⁶Of course, each individual will have different expectations, so Equation A.13 cannot hold for all individuals. However, financial economists generally speak of a *consensus* expectation. This is the expectation of the market as a whole.

Strategy 1 has no risk because the investor knows that the rate of return must be r_1 . Conversely, strategy 2 has much risk: The final return is dependent on what happens to interest rates.

Because strategy 2 has more risk than strategy 1, no risk-averse investor will choose strategy 2 if both strategies have the same expected return. Risk-averse investors can have no preference for one strategy over the other only when the expected return on strategy 2 is *above* the return on strategy 1. Because the two strategies have the same expected return when f_2 equals the spot rate expected over year 2, strategy 2 can have a higher rate of return only when the following condition holds:

Liquidity Preference Hypothesis:

$$f_2 > \text{Spot rate expected over year 2} \quad (\text{A.15})$$

That is, to induce investors to hold the riskier two-year bonds, the market sets the forward rate over the second year to be above the spot rate expected over the second year. Equation A.15 is called the *liquidity preference hypothesis*.

We developed the entire discussion by assuming that individuals are planning to invest over one year. We pointed out that for these types of individuals, a two-year bond has extra risk because it must be sold prematurely. What about individuals who want to invest for two years? (We call these people investors with a two-year *time horizon*.)

They could choose one of the following strategies:

3. Buy a two-year zero coupon bond.
4. Buy a one-year bond. When the bond matures, immediately buy another one-year bond.

Strategy 3 has no risk for an investor with a two-year time horizon because the proceeds to be received at date 2 are known as of date 0. However, strategy 4 has risk because the spot rate over year 2 is unknown at date 0. It can be shown that risk-averse investors will prefer neither strategy 3 nor strategy 4 over the other when:

$$f_2 < \text{Spot rate expected over year 2} \quad (\text{A.16})$$

Note that the assumption of risk aversion gives contrary predictions. Relationship A.15 holds for a market dominated by investors with a one-year time horizon. Relationship A.16 holds for a market dominated by investors with a two-year time horizon. Financial economists have generally argued that the time horizon of the typical investor is generally much shorter than the maturity of typical bonds in the marketplace. Thus, economists view A.15 as the best depiction of equilibrium in the bond market with *risk-averse* investors.

However, do we have a market of risk-neutral or risk-averse investors? In other words, can the expectations hypothesis of Equation A.14 or the liquidity preference hypothesis of Equation A.15 be expected to hold? As we will learn later in this book, economists view investors as being risk-averse for the most part. Yet, economists are never satisfied with a casual examination of a theory's assumptions. To them, empirical evidence of a theory's predictions must be the final arbiter.

There has been a great deal of empirical evidence about the term structure of interest rates. Unfortunately (perhaps fortunately for some students), we will not be able to present the evidence in any detail. Suffice it to say that, in our opinion, the evidence supports the liquidity preference hypothesis over the expectations hypothesis. One simple result might give students the flavor of this research. Consider an individual choosing between one of the following two strategies:

1. Invest in a one-year bond.
- 2'. Invest in a 20-year bond but sell at the end of one year.

[Strategy 2' is identical to strategy 2, except that a 20-year bond is substituted for a 2-year bond.]

The expectations hypothesis states that the expected returns on both strategies are identical. The liquidity preference hypothesis states that the expected return on strategy 2' should be above the expected return on strategy 1. Though no one knows what returns are actually expected over a particular time period, actual returns from the past may allow us to infer expectations. The results from January 1926 to December 1999 are illuminating. The average yearly return on strategy 1 is 3.8 percent and 5.5 percent on strategy 2' over this time period.^{7,8} This evidence is generally considered to be consistent with the liquidity preference hypothesis and inconsistent with the expectations hypothesis.

Questions and Problems

- Bond Pricing** The one-year spot rate is 8 percent and the two-year spot rate is 10 percent.
 - What is the price of a two-year bond that pays an annual coupon of 6 percent?
 - What is the yield to maturity of this bond?
- Bond Pricing** The one-year spot rate is 11 percent and the two-year spot rate is 8 percent. What is the price of a two-year bond that pays an annual coupon of 5 percent?
- Forward Rates** If the one-year spot rate is 7 percent and the two-year spot rate is 8.5 percent, what is the one-year forward rate over the second year?
- Forward Rates** Assume the following spot rates:

Year	Spot Rate (%)
1	4
2	5.5
3	6.5

- Calculate the one-year forward rate over the second year.
 - Calculate the one-year forward rate over the third year.
- Term Structure** Assume the following forward rates:

Maturity	Forward Rates
1	4.5%
2	6.0%

Compute the spot rates for years 1 and 2.

- Term Structure** Given the following two scenarios, for what range of spot rates expected over year 2 would you be better off adopting strategy 1? Explain.
Strategy 1: Buy a two-year bond and then sell it in year 1.
Strategy 2: Buy a one-year bond.

⁷Taken from *Stocks, Bonds, Bills and Inflation 2000 Yearbook* (Chicago: Ibbotson Associates, Inc.). Ibbotson Associates annually updates work by Roger G. Ibbotson and Rex A. Sinquefeld.

⁸It is important to note that strategy 2' does not involve buying a 20-year bond and holding it to maturity. Rather, it consists of buying a 20-year bond and selling it 1 year later—that is, when it has become a 19-year bond. This round-trip transaction occurs 74 times in the 74-year sample from January 1926 to December 1999.