CHAPTER 6

# Net Present Value and Other Investment Rules

In 2005, the automobile market in North America faced chronic overcapacity. By some estimates, General Motors may have had as many as 15 factories more than it needed. But not all automobile manufacturers faced this problem. For example, Toyota Motors announced plans for its seventh North American assembly plant, and then began a search for a site to accommodate its eighth North American plant. Each plant represents an investment of $1 billion or more. For example, Toyota's truck factory in southern Indiana was built at a cost of $2.5 billion.

Toyota's new plants are an example of a capital budgeting decision. Decisions such as these, with a price tag of over $1 billion each, are obviously major undertakings, and the risks and rewards must be carefully weighed. In this chapter, we discuss the basic tools used in making such decisions.

In Chapter 1, we saw that increasing the value of the stock in a company is the goal of financial management. Thus, what we need to know is how to tell whether a particular investment will achieve that. This chapter considers a variety of techniques that are used in practice for this purpose. More important, it shows how many of these techniques can be misleading, and it explains why the net present value approach is the right one.

## 6.1 Why Use Net Present Value?

This chapter, as well as the next two, focuses on *capital budgeting*, the decision-making process for accepting or rejecting projects. This chapter develops the basic capital budgeting methods, leaving much of the practical application to subsequent chapters. But we don't have to develop these methods from scratch. In Chapter 4, we pointed out that a dollar received in the future is worth less than a dollar received today. The reason, of course, is that today's dollar can be reinvested, yielding a greater amount in the future. And we showed in Chapter 4 that the exact worth of a dollar to be received in the future is its present value. Furthermore, Section 4.1 suggested calculating the *net present value* of any project. That is, the section suggested calculating the difference between the sum of the present values of the project's future cash flows and the initial cost of the project.

> Find out more about capital budgeting for small businesses at **www.missouribusiness.net**.

The net present value (NPV) method is the first one to be considered in this chapter. We begin by reviewing the approach with a simple example. Then, we ask why the method leads to good decisions.

EXAMPLE 6.1

**Net Present Value** The Alpha Corporation is considering investing in a riskless project costing \$100. The project receives \$107 in one year and has no other cash flows. The discount rate is 6 percent.

The NPV of the project can easily be calculated as

$$\$.94 = -\$100 + \frac{\$107}{1.06} \tag{6.1}$$

From Chapter 4, we know that the project should be accepted because its NPV is positive. Had the NPV of the project been negative, as would have been the case with an interest rate greater than 7 percent, the project should be rejected.

The basic investment rule can be generalized thus:

Accept a project if the NPV is greater than zero.

Reject a project if NPV is less than zero.

We refer to this as the **NPV rule**.

Why does the NPV rule lead to good decisions? Consider the following two strategies available to the managers of Alpha Corporation:

1. Use \$100 of corporate cash to invest in the project. The \$107 will be paid as a dividend in one year.
2. Forgo the project and pay the \$100 of corporate cash as a dividend today.

If strategy 2 is employed, the stockholder might deposit the dividend in a bank for one year. With an interest rate of 6 percent, strategy 2 would produce cash of \$106 ($= \$100 \times 1.06$) at the end of the year. The stockholder would prefer strategy 1 because strategy 2 produces less than \$107 at the end of the year.

Our basic point is this: Accepting positive NPV projects benefits the stockholders.

How do we interpret the exact NPV of \$0.94? This is the increase in the value of the firm from the project. For example, imagine that the firm today has productive assets worth \$V and has \$100 of cash. If the firm forgoes the project, the value of the firm today would simply be:

$$\$V + \$100$$

If the firm accepts the project, the firm will receive \$107 in one year but will have no cash today. Thus, the firm's value today would be:

$$\$V + \frac{\$107}{1.06}$$

The difference between these equations is just \$0.94, the present value of Equation 6.1. Thus: The value of the firm rises by the NPV of the project.

Note that the value of the firm is merely the sum of the values of the different projects, divisions, or other entities within the firm. This property, called **value additivity**, is quite important. It implies that the contribution of any project to a firm's value is simply the NPV of the project. As we will see later, alternative methods discussed in this chapter do not generally have this nice property.

One detail remains. We assumed that the project was riskless, a rather implausible assumption. Future cash flows of real-world projects are invariably risky. In other words, cash flows can only be estimated, rather than known. Imagine that the managers of Alpha *expect* the cash flow of the project to be \$107 next year. That is, the cash flow could be higher, say \$117, or lower, say \$97. With this slight change, the project is risky. Suppose the project is about as risky as the stock market as a whole, where the expected return this year is perhaps 10 percent. Then 10 percent becomes the discount rate, implying that the NPV of the project would be:

$$-\$2.73 = -\$100 + \frac{\$107}{1.10}$$

Because the NPV is negative, the project should be rejected. This makes sense: A stockholder of Alpha receiving a \$100 dividend today could invest it in the stock market, expecting a 10 percent return. Why accept a project with the same risk as the market but with an expected return of only 7 percent?

Conceptually, the discount rate on a risky project is the return that one can expect to earn on a financial asset of comparable risk. This discount rate is often referred to as an *opportunity cost* because corporate investment in the project takes away the stockholder's opportunity to invest the dividend in a financial asset. If the actual calculation of the discount rate strikes you as extremely difficult in the real world, you are probably right. Although you can call a bank to find out the current interest rate, whom do you call to find the expected return on the market this year? And, if the risk of the project differs from that of the market, how do you make the adjustment? However, the calculation is by no means impossible. We forgo the calculation in this chapter, but we present it in later chapters of the text.

Having shown that NPV is a sensible approach, how can we tell whether alternative methods are as good as NPV? The key to NPV is its three attributes:

1. *NPV uses cash flows.* Cash flows from a project can be used for other corporate purposes (such as dividend payments, other capital budgeting projects, or payments of corporate interest). By contrast, earnings are an artificial construct. Although earnings are useful to accountants, they should not be used in capital budgeting because they do not represent cash.
2. *NPV uses all the cash flows of the project.* Other approaches ignore cash flows beyond a particular date; beware of these approaches.
3. *NPV discounts the cash flows properly.* Other approaches may ignore the time value of money when handling cash flows. Beware of these approaches as well.

# 6.2 The Payback Period Method

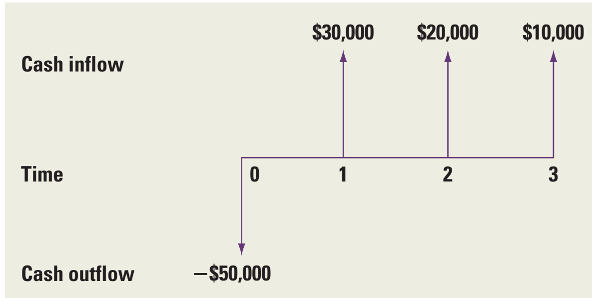
## Defining the Rule

One of the most popular alternatives to NPV is **payback**. Here is how payback works: Consider a project with an initial investment of −\$50,000. Cash flows are \$30,000, \$20,000, and \$10,000 in the first three years, respectively. These flows are illustrated in Figure 6.1. A useful way of writing down investments like the preceding is with the notation

(−\$50,000, \$30,000, \$20,000, \$10,000)

## Figure 6.1

**Cash Flows of an Investment Project**




The minus sign in front of the $50,000 reminds us that this is a cash outflow for the investor, and the commas between the different numbers indicate that they are received—or if they are cash outflows, that they are paid out—at different times. In this example we are assuming that the cash flows occur one year apart, with the first one occurring the moment we decide to take on the investment.

The firm receives cash flows of $30,000 and $20,000 in the first two years, which add up to the $50,000 original investment. This means that the firm has recovered its investment within two years. In this case two years is the *payback period* of the investment.

The **payback period rule** for making investment decisions is simple. A particular cutoff date, say two years, is selected. All investment projects that have payback periods of two years or less are accepted, and all of those that pay off in more than two years—if at all—are rejected.

## Problems with the Payback Method

There are at least three problems with payback. To illustrate the first two problems, we consider the three projects in Table 6.1. All three projects have the same three-year payback period, so they should all be equally attractive—right?

Actually, they are not equally attractive, as can be seen by a comparison of different *pairs* of projects.

**Problem 1: Timing of Cash Flows within the Payback Period** Let us compare project *A* with project *B*. In years 1 through 3, the cash flows of project *A* rise from $20 to $50, while the cash flows of project *B* fall from $50 to $20. Because the large cash flow of $50 comes earlier with project *B*, its net present value must be higher. Nevertheless, we just saw that the payback periods of the two projects are identical. Thus, a problem with the payback method is that it does not consider the timing of the cash flows within the payback period. This example shows that the payback method is inferior to NPV because, as we pointed out earlier, the NPV method *discounts the cash flows properly.*

**Problem 2: Payments after the Payback Period** Now consider projects *B* and *C*, which have identical cash flows within the payback period. However, project *C* is clearly preferred because it has a cash flow of $60,000 in the fourth year. Thus, another problem

## Table 6.1

**Expected Cash Flows for Projects *A* through *C* ($)**

| Year | *A* | *B* | *C* |
|---|---|---|---|
| 0 | −$100 | −$100 | −$100 |
| 1 | 20 | 50 | 50 |
| 2 | 30 | 30 | 30 |
| 3 | 50 | 20 | 20 |
| 4 | 60 | 60 | 60,000 |
| Payback period (years) | 3 | 3 | 3 |

with the payback method is that it ignores all cash flows occurring after the payback period. Because of the short-term orientation of the payback method, some valuable long-term projects are likely to be rejected. The NPV method does not have this flaw because, as we pointed out earlier, this method *uses all the cash flows of the project.*

**Problem 3: Arbitrary Standard for Payback Period** We do not need to refer to Table 6.1 when considering a third problem with the payback method. Capital markets help us estimate the discount rate used in the NPV method. The riskless rate, perhaps proxied by the yield on a Treasury instrument, would be the appropriate rate for a riskless investment. Later chapters of this textbook show how to use historical returns in the capital markets to estimate the discount rate for a risky project. However, there is no comparable guide for choosing the payback cutoff date, so the choice is somewhat arbitrary.

## *Managerial Perspective*

The payback method is often used by large, sophisticated companies when making relatively small decisions. The decision to build a small warehouse, for example, or to pay for a tune-up for a truck is the sort of decision that is often made by lower-level management. Typically, a manager might reason that a tune-up would cost, say, \$200, and if it saved \$120 each year in reduced fuel costs, it would pay for itself in less than two years. On such a basis the decision would be made.

Although the treasurer of the company might not have made the decision in the same way, the company endorses such decision making. Why would upper management condone or even encourage such retrograde activity in its employees? One answer would be that it is easy to make decisions using payback. Multiply the tune-up decision into 50 such decisions a month, and the appeal of this simple method becomes clearer.

The payback method also has some desirable features for managerial control. Just as important as the investment decision itself is the company's ability to evaluate the manager's decision-making ability. Under the NPV method, a long time may pass before one decides whether a decision was correct. With the payback method we know in two years whether the manager's assessment of the cash flows was correct.

It has also been suggested that firms with good investment opportunities but no available cash may justifiably use payback. For example, the payback method could be used by small, privately held firms with good growth prospects but limited access to the capital markets. Quick cash recovery enhances the reinvestment possibilities for such firms.

Finally, practitioners often argue that standard academic criticisms of payback overstate any real-world problems with the method. For example, textbooks typically make fun of payback by positing a project with low cash inflows in the early years but a huge cash inflow right after the payback cutoff date. This project is likely to be rejected under the payback method, though its acceptance would, in truth, benefit the firm. Project *C* in our Table 6.1 is an example of such a project. Practitioners point out that the pattern of cash flows in these textbook examples is much too stylized to mirror the real world. In fact, a number of executives have told us that for the overwhelming majority of real-world projects, both payback and NPV lead to the same decision. In addition, these executives indicate that if an investment like project *C* were encountered in the real world, decision makers would almost certainly make ad hoc adjustments to the payback rule so that the project would be accepted.

Notwithstanding all of the preceding rationale, it is not surprising to discover that as the decisions grow in importance, which is to say when firms look at bigger projects, NPV becomes the order of the day. When questions of controlling and evaluating the manager become less important than making the right investment decision, payback is used less frequently. For big-ticket decisions, such as whether or not to buy a machine, build a factory, or acquire a company, the payback method is seldom used.

### Summary of Payback

The payback method differs from NPV and is therefore conceptually wrong. With its arbitrary cutoff date and its blindness to cash flows after that date, it can lead to some flagrantly foolish decisions if it is used too literally. Nevertheless, because of its simplicity, as well as its other mentioned advantages, companies often use it as a screen for making the myriad minor investment decisions they continually face.

Although this means that you should be wary of trying to change approaches such as the payback method when you encounter them in companies, you should probably be careful not to accept the sloppy financial thinking they represent. After this course, you would do your company a disservice if you used payback instead of NPV when you had a choice.

## 6.3 The Discounted Payback Period Method

Aware of the pitfalls of payback, some decision makers use a variant called the **discounted payback period method**. Under this approach, we first discount the cash flows. Then we ask how long it takes for the discounted cash flows to equal the initial investment.

For example, suppose that the discount rate is 10 percent and the cash flows on a project are given by:

$$(-\$100, \$50, \$50, \$20)$$

This investment has a payback period of two years because the investment is paid back in that time.

To compute the project's discounted payback period, we first discount each of the cash flows at the 10 percent rate. These discounted cash flows are:

$$[-\$100, \$50/1.1, \$50/(1.1)^2, \$20/(1.1)^3] = (-\$100, \$45.45, \$41.32, \$15.03)$$

The discounted payback period of the original investment is simply the payback period for these discounted cash flows. The payback period for the discounted cash flows is slightly less than three years because the discounted cash flows over the three years are \$101.80 (=\$45.45 + 41.32 + 15.03). As long as the cash flows and discount rate are positive, the discounted payback period will never be smaller than the payback period because discounting reduces the value of the cash flows.

At first glance discounted payback may seem like an attractive alternative, but on closer inspection we see that it has some of the same major flaws as payback. Like payback, discounted payback first requires us to make a somewhat magical choice of an arbitrary cutoff period, and then it ignores all cash flows after that date.

If we have already gone to the trouble of discounting the cash flows, any small appeal to simplicity or to managerial control that payback may have has been lost. We might just as well add up all the discounted cash flows and use NPV to make the decision. Although discounted payback looks a bit like NPV, it is just a poor compromise between the payback method and NPV.

## 6.4 The Average Accounting Return Method

### Defining the Rule

Another attractive, but fatally flawed, approach to financial decision making is the **average accounting return**. The average accounting return is the average project earnings after taxes and depreciation, divided by the average book value of the investment during its life.

In spite of its flaws, the average accounting return method is worth examining because it is used frequently in the real world.

EXAMPLE 6.2

**Average Accounting Return** Consider a company that is evaluating whether to buy a store in a new mall. The purchase price is $500,000. We will assume that the store has an estimated life of five years and will need to be completely scrapped or rebuilt at the end of that time. The projected yearly sales and expense figures are shown in Table 6.2.

**Table 6.2** **Projected Yearly Revenue and Costs for Average Accounting Return**

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---|---|---|---|---|
| Revenue | $433,333 | $450,000 | $266,667 | $200,000 | $133,333 |
| Expenses | 200,000 | 150,000 | 100,000 | 100,000 | 100,000 |
| Before-tax cash flow | 233,333 | 300,000 | 166,667 | 100,000 | 33,333 |
| Depreciation | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 |
| Earnings before taxes | 133,333 | 200,000 | 66,667 | 0 | − 66,667 |
| Taxes ($t_c = .25$)* | 33,333 | 50,000 | 16,667 | 0 | − 16,667 |
| Net income | $100,000 | $150,000 | $ 50,000 | $ 0 | −$ 50,000 |

$$\text{Average net income} = \frac{(\$100{,}000 + 150{,}000 + 50{,}000 + 0 - 50{,}000)}{5} = \$50{,}000$$

$$\text{Average investment} = \frac{\$500{,}000 + 0}{2} = \$250{,}000$$

$$\text{AAR} = \frac{\$50{,}000}{\$250{,}000} = 20\%$$

*Corporate tax rate = $t_c$. The tax rebate in year 5 of −$16,667 occurs if the rest of the firm is profitable. Here the loss in the project reduces the taxes of the entire firm.

It is worth examining Table 6.2 carefully. In fact, the first step in any project assessment is a careful look at projected cash flows. First-year sales for the store are estimated to be $433,333. Before-tax cash flow will be $233,333. Sales are expected to rise and expenses are expected to fall in the second year, resulting in a before-tax cash flow of $300,000. Competition from other stores and the loss in novelty will reduce before-tax cash flow to $166,667, $100,000, and $33,333, respectively, in the next three years.

To compute the average accounting return (AAR) on the project, we divide the average net income by the average amount invested. This can be done in three steps.

**Step 1: Determining Average Net Income** Net income in any year is net cash flow minus depreciation and taxes. Depreciation is *not* a cash outflow.[1] Rather, it is a charge reflecting the fact that the investment in the store becomes less valuable every year.

We assume the project has a useful life of five years, at which time it will be worthless. Because the initial investment is $500,000 and because it will be worthless in five years, we assume that it loses value at the rate of $100,000 each year. This steady loss in value of $100,000 is called *straight-line depreciation*. We subtract both depreciation and taxes from before-tax cash flow to derive net income, as shown in Table 6.2. Net income is $100,000

[1]Depreciation will be treated in more detail in the next chapter.

in the first year, \$150,000 in year 2, \$50,000 in year 3, zero in year 4, and −\$50,000 in the last year. The average net income over the life of the project is therefore:

**Average Net Income**:

$$[\$100{,}000 + 150{,}000 + 50{,}000 + 0 + (-50{,}000)]/5 = \$50{,}000$$

**Step 2: Determining Average Investment** We stated earlier that, due to depreciation, the investment in the store becomes less valuable every year. Because depreciation is \$100,000 per year, the value at the end of year zero is \$500,000, the value at the end of year 1 is \$400,000, and so on. What is the average value of the investment over the life of the investment?

The mechanical calculation is:

**Average Investment**:

$$\begin{aligned}&(\$500{,}000 + 400{,}000 + 300{,}000 + 200{,}000 + 100{,}000 + 0)/6 \\ &\quad = \$250{,}000\end{aligned} \qquad \textbf{(6.2)}$$

We divide by 6, not 5, because \$500,000 is what the investment is worth at the beginning of the five years and \$0 is what it is worth at the beginning of the sixth year. In other words, there are six terms in the parentheses of Equation 6.2.

**Step 3: Determining AAR** The average return is simply:

$$\text{AAR} = \frac{\$50{,}000}{\$250{,}000} = 20\%$$

If the firm had a targeted accounting rate of return greater than 20 percent, the project would be rejected; if its targeted return were less than 20 percent, it would be accepted.

## *Analyzing the Average Accounting Return Method*

By now you should be able to see what is wrong with the AAR method.

The most important flaw with AAR is that it does not work with the right raw materials. It uses net income and book value of the investment, both of which come from the accounting books. Accounting numbers are somewhat arbitrary. For example, certain cash outflows, such as the cost of a building, are depreciated under current accounting rules. Other flows, such as maintenance, are expensed. In real-world situations, the decision to depreciate or expense an item involves judgment. Thus, the basic inputs of the AAR method, income and average investment, are affected by the accountant's judgment. Conversely, the NPV method *uses cash flows*. Accounting judgments do not affect cash flow.

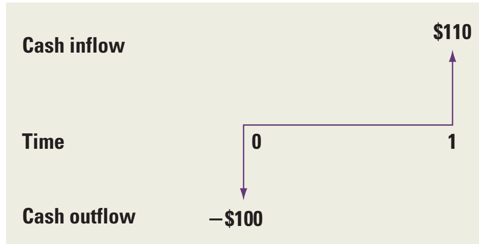
Second, AAR takes no account of timing. In the previous example, the AAR would have been the same if the \$100,000 net income in the first year had occurred in the last year. However, delaying an inflow for five years would have lowered the NPV of the investment. As mentioned earlier in this chapter, the NPV approach *discounts properly*.

Third, just as payback requires an arbitrary choice of the cutoff date, the AAR method offers no guidance on what the right targeted rate of return should be. It could be the discount rate in the market. But then again, because the AAR method is not the same as the present value method, it is not obvious that this would be the right choice.

Given these problems, is the AAR method employed in practice? Like the payback method, the AAR (and variations of it) is frequently used as a "backup" to discounted cash flow methods. Perhaps this is so because it is easy to calculate and uses accounting numbers readily available from the firm's accounting system. In addition, both stockholders and the media pay a lot of attention to the overall profitability of a firm. Thus, some managers

**Figure 6.2**

**Cash Flows for a Simple Project**




may feel pressured to select projects that are profitable in the near term, even if the projects come up short in terms of NPV. These managers may focus on the AAR of individual projects more than they should.

## 6.5 The Internal Rate of Return

Now we come to the most important alternative to the NPV method: the internal rate of return, universally known as the IRR. The IRR is about as close as you can get to the NPV without actually being the NPV. The basic rationale behind the IRR method is that it provides a single number summarizing the merits of a project. That number does not depend on the interest rate prevailing in the capital market. That is why it is called the internal rate of return; the number is internal or intrinsic to the project and does not depend on anything except the cash flows of the project.

For example, consider the simple project (−\$100, \$110) in Figure 6.2. For a given rate, the net present value of this project can be described as:

$$\text{NPV} = -\$100 + \frac{\$110}{1 + R}$$

where $R$ is the discount rate. What must the discount rate be to make the NPV of the project equal to zero?

We begin by using an arbitrary discount rate of .08, which yields:

$$\$1.85 = -\$100 + \frac{\$110}{1.08}$$

Because the NPV in this equation is positive, we now try a higher discount rate, such as .12. This yields:

$$-\$1.79 = -\$100 + \frac{\$110}{1.12}$$

Because the NPV in this equation is negative, we try lowering the discount rate to .10. This yields:

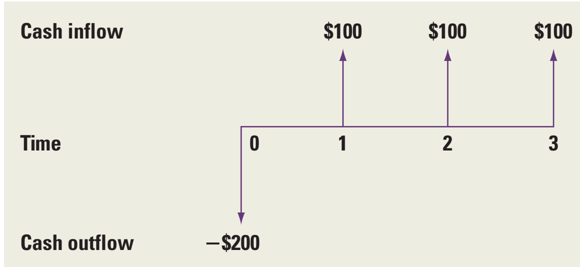
$$0 = -\$100 + \frac{\$110}{1.10}$$

This trial-and-error procedure tells us that the NPV of the project is zero when $R$ equals 10 percent.[2] Thus, we say that 10 percent is the project's **internal rate of return** (IRR). In

[2]Of course, we could have directly solved for $R$ in this example after setting NPV equal to zero. However, with a long series of cash flows, one cannot generally directly solve for $R$. Instead, one is forced to use trial and error (or let a machine use trial and error).

**Figure 6.3**

**Cash Flows for a More Complex Project**




general, the IRR is the rate that causes the NPV of the project to be zero. The implication of this exercise is very simple. The firm should be equally willing to accept or reject the project if the discount rate is 10 percent. The firm should accept the project if the discount rate is below 10 percent. The firm should reject the project if the discount rate is above 10 percent.

The general investment rule is clear:

**Accept the project if the IRR is greater than the discount rate. Reject the project if the IRR is less than the discount rate**.

We refer to this as the **basic IRR rule**. Now we can try the more complicated example (−$200, $100, $100, $100) in Figure 6.3.

As we did previously, let's use trial and error to calculate the internal rate of return. We try 20 percent and 30 percent, yielding the following:

| Discount Rate | NPV |
|---|---|
| 20% | $10.65 |
| 30 | −18.39 |

After much more trial and error, we find that the NPV of the project is zero when the discount rate is 23.37 percent. Thus, the IRR is 23.37 percent. With a 20 percent discount rate, the NPV is positive and we would accept it. However, if the discount rate were 30 percent, we would reject it.

Algebraically, IRR is the unknown in the following equation:[3]

$$0 = -\$200 + \frac{\$100}{1 + \text{IRR}} + \frac{\$100}{(1 + \text{IRR})^2} + \frac{\$100}{(1 + \text{IRR})^3}$$

Figure 6.4 illustrates what the IRR of a project means. The figure plots the NPV as a function of the discount rate. The curve crosses the horizontal axis at the IRR of 23.37 percent because this is where the NPV equals zero.

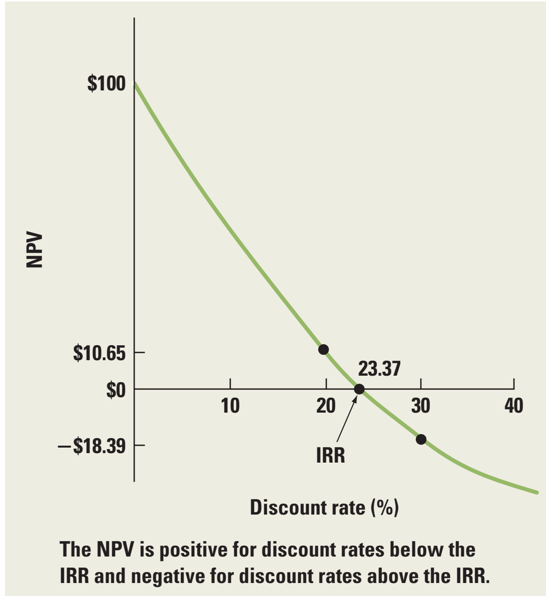
It should also be clear that the NPV is positive for discount rates below the IRR and negative for discount rates above the IRR. This means that if we accept projects like this one when the discount rate is less than the IRR, we will be accepting positive NPV projects. Thus, the IRR rule coincides exactly with the NPV rule.

If this were all there were to it, the IRR rule would always coincide with the NPV rule. This would be a wonderful discovery because it would mean that just by computing the

[3]One can derive the IRR directly for a problem with an initial outflow and up to four subsequent inflows. In the case of two subsequent inflows, for example, the quadratic formula is needed. In general, however, only trial and error will work for an outflow and five or more subsequent inflows.

**Figure 6.4**

**Net Present Value (NPV) and Discount Rates for a More Complex Project**




IRR for a project we would be able to tell where it ranks among all of the projects we are considering. For example, if the IRR rule really works, a project with an IRR of 20 percent will always be at least as good as one with an IRR of 15 percent.

But the world of finance is not so kind. Unfortunately, the IRR rule and the NPV rule are the same only for examples like the one just discussed. Several problems with the IRR approach occur in more complicated situations.

# 6.6 Problems with the IRR Approach

## Definition of Independent and Mutually Exclusive Projects

An **independent project** is one whose acceptance or rejection is independent of the acceptance or rejection of other projects. For example, imagine that McDonald's is considering putting a hamburger outlet on a remote island. Acceptance or rejection of this unit is likely to be unrelated to the acceptance or rejection of any other restaurant in its system. The remoteness of the outlet in question ensures that it will not pull sales away from other outlets.

Now consider the other extreme, **mutually exclusive investments**. What does it mean for two projects, *A* and *B*, to be mutually exclusive? You can accept *A* or you can accept *B* or you can reject both of them, but you cannot accept both of them. For example, *A* might be a decision to build an apartment house on a corner lot that you own, and *B* might be a decision to build a movie theater on the same lot.

We now present two general problems with the IRR approach that affect both independent and mutually exclusive projects. Then we deal with two problems affecting mutually exclusive projects only.

## Two General Problems Affecting Both Independent and Mutually Exclusive Projects

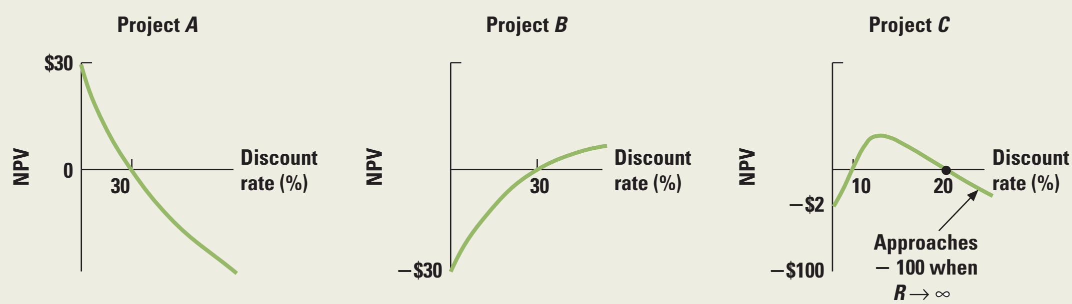
We begin our discussion with project *A*, which has the following cash flows:

$$(-\$100, \$130)$$

**Table 6.3** The Internal Rate of Return and Net Present Value

| | Project *A* | | | Project *B* | | | Project *C* | | |
|---|---|---|---|---|---|---|---|---|---|
| **Dates:** | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 |
| Cash flows | −$100 | $130 | | $100 | −$130 | | −$100 | $230 | −$132 |
| IRR | | 30% | | | 30% | | 10% | and | 20% |
| NPV @10% | | $18.2 | | | −$18.2 | | | 0 | |
| Accept if market rate | | <30% | | | >30% | | >10% | but | <20% |
| Financing or investing | | Investing | | | Financing | | | Mixture | |

**Figure 6.5** Net Present Value and Discount Rates for Projects *A*, *B*, and *C*




Project *A* has a cash outflow at date 0 followed by a cash inflow at date 1. Its NPV is negatively related to the discount rate. Project *B* has a cash inflow at date 0 followed by a cash outflow at date 1. Its NPV is positively related to the discount rate. Project *C* has two changes of sign in its cash flows. It has an outflow at date 0, an inflow at date 1, and an outflow at date 2. Projects with more than one change of sign can have multiple rates of return.

The IRR for project *A* is 30 percent. Table 6.3 provides other relevant information about the project. The relationship between NPV and the discount rate is shown for this project in Figure 6.5. As you can see, the NPV declines as the discount rate rises.

**Problem 1: Investing or Financing?** Now consider project *B*, with cash flows of:

$$(\$100, -\$130)$$

These cash flows are exactly the reverse of the flows for project *A*. In project *B*, the firm receives funds first and then pays out funds later. While unusual, projects of this type do exist. For example, consider a corporation conducting a seminar where the participants pay in advance. Because large expenses are frequently incurred at the seminar date, cash inflows precede cash outflows.

Consider our trial-and-error method to calculate IRR:

$$-\$4 = +\$100 - \frac{\$130}{1.25}$$

$$\$0 = +\$100 - \frac{\$130}{1.30}$$

$$\$3.70 = +\$100 - \frac{\$130}{1.35}$$

As with project *A*, the internal rate of return is 30 percent. However, notice that the net present value is *negative* when the discount rate is *below* 30 percent. Conversely, the net present value is positive when the discount rate is above 30 percent. The decision rule is exactly the opposite of our previous result. For this type of a project, the following rule applies:

**Accept the project when the IRR is less than the discount rate. Reject the project when the IRR is greater than the discount rate.**

This unusual decision rule follows from the graph of project *B* in Figure 6.5. The curve is upward sloping, implying that NPV is *positively* related to the discount rate.

The graph makes intuitive sense. Suppose the firm wants to obtain $100 immediately. It can either (1) accept project *B* or (2) borrow $100 from a bank. Thus, the project is actually a substitute for borrowing. In fact, because the IRR is 30 percent, taking on project *B* is tantamount to borrowing at 30 percent. If the firm can borrow from a bank at, say, only 25 percent, it should reject the project. However, if a firm can borrow from a bank only at, say, 35 percent, it should accept the project. Thus project *B* will be accepted if and only if the discount rate is *above* the IRR.[4]

This should be contrasted with project *A*. If the firm has $100 of cash to invest, it can either (1) accept project *A* or (2) lend $100 to the bank. The project is actually a substitute for lending. In fact, because the IRR is 30 percent, taking on project *A* is tantamount to lending at 30 percent. The firm should accept project *A* if the lending rate is below 30 percent. Conversely, the firm should reject project *A* if the lending rate is above 30 percent.

Because the firm initially pays out money with project *A* but initially receives money with project *B*, we refer to project *A* as an *investing type project* and project *B* as a *financing type project*. Investing type projects are the norm. Because the IRR rule is reversed for financing type projects, be careful when using it with this type of project.

**Problem 2: Multiple Rates of Return** Suppose the cash flows from a project are:

$$(-\$100, \$230, -\$132)$$

Because this project has a negative cash flow, a positive cash flow, and another negative cash flow, we say that the project's cash flows exhibit two changes of sign, or "flip-flops." Although this pattern of cash flows might look a bit strange at first, many projects require outflows of cash after receiving some inflows. An example would be a strip-mining project. The first stage in such a project is the initial investment in excavating the mine. Profits from operating the mine are received in the second stage. The third stage involves a further investment to reclaim the land and satisfy the requirements of environmental protection legislation. Cash flows are negative at this stage.

Projects financed by lease arrangements may produce a similar pattern of cash flows. Leases often provide substantial tax subsidies, generating cash inflows after an initial investment. However, these subsidies decline over time, frequently leading to negative cash flows in later years. (The details of leasing will be discussed in a later chapter.)

It is easy to verify that this project has not one but two IRRs, 10 percent and 20 percent.[5] In a case like this, the IRR does not make any sense. What IRR are we to use—10 percent

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[4]This paragraph implicitly assumes that the cash flows of the project are risk-free. In this way we can treat the borrowing rate as the discount rate for a firm needing $100. With risky cash flows, another discount rate would be chosen. However, the intuition behind the decision to accept when the IRR is less than the discount rate would still apply.

[5]The calculations are

$$-\$100 + \frac{\$230}{1.1} - \frac{\$132}{(1.1)^2}$$
$$-\$100 + 209.09 - 109.09 = 0$$

and

$$-\$100 + \frac{\$230}{1.2} - \frac{\$132}{(1.2)^2}$$
$$-\$100 + 191.67 - 91.67 = 0$$

Thus, we have multiple rates of return.

or 20 percent? Because there is no good reason to use one over the other, IRR simply cannot be used here.

Why does this project have multiple rates of return? Project *C* generates multiple internal rates of return because both an inflow and an outflow occur after the initial investment. In general, these flip-flops or changes in sign produce multiple IRRs. In theory, a cash flow stream with *K* changes in sign can have up to *K* sensible internal rates of return (IRRs above −100 percent). Therefore, because project *C* has two changes in sign, it can have as many as two IRRs. As we pointed out, projects whose cash flows change sign repeatedly can occur in the real world.

**NPV Rule** Of course, we should not be too worried about multiple rates of return. After all, we can always fall back on the NPV rule. Figure 6.5 plots the NPV of project *C* (−$100, $230, −$132) as a function of the discount rate. As the figure shows, the NPV is zero at both 10 percent and 20 percent and negative outside the range. Thus, the NPV rule tells us to accept the project if the appropriate discount rate is between 10 percent and 20 percent. The project should be rejected if the discount rate lies outside this range.

**Modified IRR** As an alternative to NPV, we now introduce the **modified IRR (MIRR)** method, which handles the multiple IRR problem by combining cash flows until only one change in sign remains. To see how it works, consider project *C* again. With a discount rate of, say, 14 percent, the value of the last cash flow, −$132, is:

$$-\$132/1.14 = -\$115.79$$

as of date 1. Because $230 is already received at that time, the "adjusted" cash flow at date 1 is $114.21 (= $230 − 115.79). Thus, the MIRR approach produces the following two cash flows for the project:

$$(-\$100, \$114.21)$$

Note that by discounting and then combining cash flows, we are left with only one change in sign. The IRR rule can now be applied. The IRR of these two cash flows is 14.21 percent, implying that the project should be accepted given our assumed discount rate of 14 percent.

Of course, project *C* is relatively simple to begin with: It has only three cash flows and two changes in sign. However, the same procedure can easily be applied to more complex projects—that is, just keep discounting and combining the later cash flows until only one change of sign remains.

Although this adjustment does correct for multiple IRRs, it appears, at least to us, to violate the "spirit" of the IRR approach. As stated earlier, the basic rationale behind the IRR method is that it provides a single number summarizing the merits of a project. That number does not depend on the discount rate. In fact, that is why it is called the internal rate of return: The number is *internal*, or intrinsic, to the project and does not depend on anything except the cash flows of the project. By contrast, MIRR is clearly a function of the discount rate. However, a firm using this adjustment will avoid the multiple IRR problem, just as a firm using the NPV rule will avoid it.

**The Guarantee against Multiple IRRs** If the first cash flow of a project is negative (because it is the initial investment) and if all of the remaining flows are positive, there can be only a single, unique IRR, no matter how many periods the project lasts. This is easy to understand by using the concept of the time value of money. For example, it is simple to verify that project *A* in Table 6.3 has an IRR of 30 percent because using a 30-percent discount rate gives

$$\begin{aligned} \text{NPV} &= -\$100 + \$130/(1.3) \\ &= \$0 \end{aligned}$$

How do we know that this is the only IRR? Suppose we were to try a discount rate greater than 30 percent. In computing the NPV, changing the discount rate does not change the value of the initial cash flow of −$100 because that cash flow is not discounted. But raising the discount rate can only lower the present value of the future cash flows. In other words, because the NPV is zero at 30 percent, any increase in the rate will push the NPV into the negative range. Similarly, if we try a discount rate of less than 30 percent, the overall NPV of the project will be positive. Though this example has only one positive flow, the above reasoning still implies a single, unique IRR if there are many inflows (but no outflows) after the initial investment.

If the initial cash flow is positive—and if all of the remaining flows are negative—there can only be a single, unique IRR. This result follows from similar reasoning. Both these cases have only one change of sign or flip-flop in the cash flows. Thus, we are safe from multiple IRRs whenever there is only one sign change in the cash flows.

**General Rules** The following chart summarizes our rules:

| Flows | Number of IRRs | IRR Criterion | NPV Criterion |
|---|---|---|---|
| First cash flow is negative and all remaining cash flows are positive. | 1 | Accept if IRR > *R*.<br>Reject if IRR < *R*. | Accept if NPV > 0.<br>Reject if NPV < 0. |
| First cash flow is positive and all remaining cash flows are negative. | 1 | Accept if IRR < *R*.<br>Reject if IRR > *R*. | Accept if NPV > 0.<br>Reject if NPV < 0. |
| Some cash flows after first are positive and some cash flows after first are negative. | May be more than 1. | No valid IRR. | Accept if NPV > 0.<br>Reject if NPV < 0. |

Note that the NPV criterion is the same for each of the three cases. In other words, NPV analysis is always appropriate. Conversely, the IRR can be used only in certain cases. When it comes to NPV, the preacher's words, "You just can't lose with the stuff I use," clearly apply.

## *Problems Specific to Mutually Exclusive Projects*

As mentioned earlier, two or more projects are mutually exclusive if the firm can accept only one of them. We now present two problems dealing with the application of the IRR approach to mutually exclusive projects. These two problems are quite similar, though logically distinct.

**The Scale Problem** A professor we know motivates class discussions of this topic with this statement: "Students, I am prepared to let one of you choose between two mutually exclusive 'business' propositions. Opportunity 1—You give me $1 now and I'll give you $1.50 back at the end of the class period. Opportunity 2—You give me $10 and I'll give you $11 back at the end of the class period. You can choose only one of the two opportunities. And you cannot choose either opportunity more than once. I'll pick the first volunteer."

Which would you choose? The correct answer is opportunity 2.[6] To see this, look at the following chart:

| | Cash Flow at Beginning of Class | Cash Flow at End of Class (90 Minutes Later) | NPV[7] | IRR |
|---|---|---|---|---|
| Opportunity 1 | −$ 1 | +$ 1.50 | $ .50 | 50% |
| Opportunity 2 | − 10 | + 11.00 | 1.00 | 10 |

As we have stressed earlier in the text, one should choose the opportunity with the highest NPV. This is opportunity 2 in the example. Or, as one of the professor's students explained it, "I'm bigger than the professor, so I know I'll get my money back. And I have $10 in my pocket right now so I can choose either opportunity. At the end of the class, I'll be able to play two rounds of my favorite electronic game with opportunity 2 and still have my original investment, safe and sound.[8] The profit on opportunity 1 buys only one round."

This business proposition illustrates a defect with the internal rate of return criterion. The basic IRR rule indicates the selection of opportunity 1 because the IRR is 50 percent. The IRR is only 10 percent for opportunity 2.

Where does IRR go wrong? The problem with IRR is that it ignores issues of *scale.* Although opportunity 1 has a greater IRR, the investment is much smaller. In other words, the high percentage return on opportunity 1 is more than offset by the ability to earn at least a decent return[9] on a much bigger investment under opportunity 2.

Because IRR seems to be misguided here, can we adjust or correct it? We illustrate how in the next example.

**EXAMPLE 6.3**

**NPV versus IRR** Stanley Jaffe and Sherry Lansing have just purchased the rights to *Corporate Finance: The Motion Picture*. They will produce this major motion picture on either a small budget or a big budget. Here are the estimated cash flows:

| | Cash Flow at Date 0 | Cash Flow at Date 1 | NPV @25% | IRR |
|---|---|---|---|---|
| Small budget | −$10 million | $40 million | $22 million | 300% |
| Large budget | − 25 million | 65 million | 27 million | 160 |

Because of high risk, a 25 percent discount rate is considered appropriate. Sherry wants to adopt the large budget because the NPV is higher. Stanley wants to adopt the small budget because the IRR is higher. Who is right?

(*continued*)

[6]The professor uses real money here. Though many students have done poorly on the professor's exams over the years, no student ever chose opportunity 1. The professor claims that his students are "money players."

[7]We assume a zero rate of interest because his class lasted only 90 minutes. It just seemed like a lot longer.

[8]At press time for this text, electronic games cost $0.50 apiece.

[9]A 10 percent return is more than decent over a 90-minute interval!

For the reasons espoused in the classroom example, NPV is correct. Hence Sherry is right. However, Stanley is very stubborn where IRR is concerned. How can Sherry justify the large budget to Stanley using the IRR approach?

This is where *incremental IRR* comes in. Sherry calculates the incremental cash flows from choosing the large budget instead of the small budget as follows:

| | Cash Flow at Date 0 (in \$ millions) | Cash Flow at Date 1 (in \$ millions) |
|---|---|---|
| Incremental cash flows from choosing large budget instead of small budget | $-\$25 - (-10) = -\$15$ | $\$65 - 40 = \$25$ |

This chart shows that the incremental cash flows are −\$15 million at date 0 and \$25 million at date 1. Sherry calculates incremental IRR as follows:

**Formula for Calculating the Incremental IRR:**

$$0 = -\$15 \text{ million} + \frac{\$25 \text{ million}}{1 + \text{IRR}}$$

IRR equals 66.67 percent in this equation, implying that the **incremental IRR** is 66.67 percent. Incremental IRR is the IRR on the incremental investment from choosing the large project instead of the small project.

In addition, we can calculate the NPV of the incremental cash flows:

**NPV of Incremental Cash Flows:**

$$-\$15 \text{ million} + \frac{\$25 \text{ million}}{1.25} = \$5 \text{ million}$$

We know the small-budget picture would be acceptable as an independent project because its NPV is positive. We want to know whether it is beneficial to invest an additional \$15 million to make the large-budget picture instead of the small-budget picture. In other words, is it beneficial to invest an additional \$15 million to receive an additional \$25 million next year? First, our calculations show the NPV on the incremental investment to be positive. Second, the incremental IRR of 66.67 percent is higher than the discount rate of 25 percent. For both reasons, the incremental investment can be justified, so the large-budget movie should be made. The second reason is what Stanley needed to hear to be convinced.

In review, we can handle this example (or any mutually exclusive example) in one of three ways:

1. *Compare the NPVs of the two choices*. The NPV of the large-budget picture is greater than the NPV of the small-budget picture. That is, \$27 million is greater than \$22 million.
2. *Calculate the incremental NPV from making the large-budget picture instead of the small-budget picture*. Because the incremental NPV equals \$5 million, we choose the large-budget picture.
3. *Compare the incremental IRR to the discount rate*. Because the incremental IRR is 66.67 percent and the discount rate is 25 percent, we take the large-budget picture.

All three approaches always give the same decision. However, we must *not* compare the IRRs of the two pictures. If we did, we would make the wrong choice. That is, we would accept the small-budget picture.

Although students frequently think that problems of scale are relatively unimportant, the truth is just the opposite. A well-known chef on TV often says, "I don't know about your flour, but the flour I buy don't come seasoned." The same thing applies to capital budgeting. No real-world project comes in one clear-cut size. Rather, the firm has to *determine* the best size for the project. The movie budget of $25 million is not fixed in stone. Perhaps an extra $1 million to hire a bigger star or to film at a better location will increase the movie's gross. Similarly, an industrial firm must decide whether it wants a warehouse of, say, 500,000 square feet or 600,000 square feet. And, earlier in the chapter, we imagined McDonald's opening an outlet on a remote island. If it does this, it must decide how big the outlet should be. For almost any project, someone in the firm has to decide on its size, implying that problems of scale abound in the real world.

One final note here. Students often ask which project should be subtracted from the other in calculating incremental flows. Notice that we are subtracting the smaller project's cash flows from the bigger project's cash flows. This leaves an *outflow* at date 0. We then use the basic IRR rule on the incremental flows.[10]

**The Timing Problem** Next we illustrate another, quite similar problem with the IRR approach to evaluating mutually exclusive projects.

EXAMPLE 6.4

**Mutually Exclusive Investments** Suppose that the Kaufold Corporation has two alternative uses for a warehouse. It can store toxic waste containers (investment *A*) or electronic equipment (investment *B*). The cash flows are as follows:

| | Cash Flow at Year | | | | NPV | | | |
|---|---|---|---|---|---|---|---|---|
| **Year:** | **0** | **1** | **2** | **3** | **@0%** | **@10%** | **@15%** | **IRR** |
| Investment *A* | −$10,000 | $10,000 | $1,000 | $ 1,000 | $2,000 | $669 | $109 | 16.04% |
| Investment *B* | −10,000 | 1,000 | 1,000 | 12,000 | 4,000 | 751 | −484 | 12.94 |

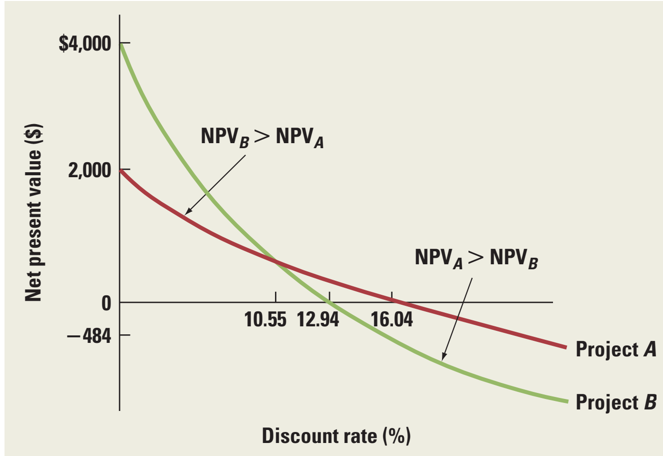
We find that the NPV of investment *B* is higher with low discount rates, and the NPV of investment *A* is higher with high discount rates. This is not surprising if you look closely at the cash flow patterns. The cash flows of *A* occur early, whereas the cash flows of *B* occur later. If we assume a high discount rate, we favor investment *A* because we are implicitly assuming that the early cash flow (for example, $10,000 in year 1) can be reinvested at that rate. Because most of investment *B*'s cash flows occur in year 3, *B*'s value is relatively high with low discount rates.

The patterns of cash flow for both projects appear in Figure 6.6. Project *A* has an NPV of $2,000 at a discount rate of zero. This is calculated by simply adding up the cash flows without discounting them. Project *B* has an NPV of $4,000 at the zero rate. However, the NPV of project *B* declines more rapidly as the discount rate increases than does the NPV

[10]Alternatively, we could have subtracted the larger project's cash flows from the smaller project's cash flows. This would have left an *inflow* at date 0, making it necessary to use the IRR rule for financing situations. This would work, but we find it more confusing.

**Figure 6.6**

**Net Present Value and the Internal Rate of Return for Mutually Exclusive Projects**




of project *A*. As we mentioned, this occurs because the cash flows of *B* occur later. Both projects have the same NPV at a discount rate of 10.55 percent. The IRR for a project is the rate at which the NPV equals zero. Because the NPV of *B* declines more rapidly, *B* actually has a lower IRR.

As with the movie example, we can select the better project with one of three different methods:

1. *Compare NPVs of the two projects.* Figure 6.6 aids our decision. If the discount rate is below 10.55 percent, we should choose project *B* because *B* has a higher NPV. If the rate is above 10.55 percent, we should choose project *A* because *A* has a higher NPV.
2. *Compare incremental IRR to discount rate.* Method 1 employed NPV. Another way of determining that *B* is a better project is to subtract the cash flows of *A* from the cash flows of *B* and then to calculate the IRR. This is the incremental IRR approach we spoke of earlier.

    Here are the incremental cash flows:

| | | | | | | NPV of Incremental Cash Flows | | |
|---|---|---|---|---|---|---|---|---|
| **Year:** | **0** | **1** | **2** | **3** | **Incremental IRR** | **@0%** | **@10%** | **@15%** |
| *B* − *A* | 0 | −$9,000 | 0 | $11,000 | 10.55% | $2,000 | $83 | −$593 |

    This chart shows that the incremental IRR is 10.55 percent. In other words, the NPV on the incremental investment is zero when the discount rate is 10.55 percent. Thus, if the relevant discount rate is below 10.55 percent, project *B* is preferred to project *A*. If the relevant discount rate is above 10.55 percent, project *A* is preferred to project *B*.[11]

[11] In this example, we first showed that the NPVs of the two projects are equal when the discount rate is 10.55 percent. We next showed that the incremental IRR is also 10.55 percent. This is not a coincidence; this equality must *always* hold. The incremental IRR is the rate that causes the incremental cash flows to have zero NPV. The incremental cash flows have zero NPV when the two projects have the same NPV.

3. *Calculate NPV on incremental cash flows.* Finally, we could calculate the NPV on the incremental cash flows. The chart that appears with the previous method displays these NPVs. We find that the incremental NPV is positive when the discount rate is either 0 percent or 10 percent. The incremental NPV is negative if the discount rate is 15 percent. If the NPV is positive on the incremental flows, we should choose *B.* If the NPV is negative, we should choose *A*.

In summary, the same decision is reached whether we (1) compare the NPVs of the two projects, (2) compare the incremental IRR to the relevant discount rate, or (3) examine the NPV of the incremental cash flows. However, as mentioned earlier, we should *not* compare the IRR of project *A* with the IRR of project *B.*

We suggested earlier that we should subtract the cash flows of the smaller project from the cash flows of the bigger project. What do we do here when the two projects have the same initial investment? Our suggestion in this case is to perform the subtraction so that the *first* nonzero cash flow is negative. In the Kaufold Corp. example we achieved this by subtracting *A* from *B.* In this way, we can still use the basic IRR rule for evaluating cash flows.

The preceding examples illustrate problems with the IRR approach in evaluating mutually exclusive projects. Both the professor–student example and the motion picture example illustrate the problem that arises when mutually exclusive projects have different initial investments. The Kaufold Corp. example illustrates the problem that arises when mutually exclusive projects have different cash flow timing. When working with mutually exclusive projects, it is not necessary to determine whether it is the scale problem or the timing problem that exists. Very likely both occur in any real-world situation. Instead, the practitioner should simply use either an incremental IRR or an NPV approach.

## Redeeming Qualities of IRR

IRR probably survives because it fills a need that NPV does not. People seem to want a rule that summarizes the information about a project in a single rate of return. This single rate gives people a simple way of discussing projects. For example, one manager in a firm might say to another, "Remodeling the north wing has a 20 percent IRR."

To their credit, however, companies that employ the IRR approach seem to understand its deficiencies. For example, companies frequently restrict managerial projections of cash flows to be negative at the beginning and strictly positive later. Perhaps, then, the ability of the IRR approach to capture a complex investment project in a single number and the ease of communicating that number explain the survival of the IRR.

## A Test

To test your knowledge, consider the following two statements:

1. You must know the discount rate to compute the NPV of a project, but you compute the IRR without referring to the discount rate.
2. Hence, the IRR rule is easier to apply than the NPV rule because you don't use the discount rate when applying IRR.

The first statement is true. The discount rate is needed to *compute* NPV. The IRR is *computed* by solving for the rate where the NPV is zero. No mention is made of the discount rate in the mere computation. However, the second statement is false. To *apply* IRR, you must compare the internal rate of return with the discount rate. Thus the discount rate is needed for making a decision under either the NPV or IRR approach.

# 6.7 The Profitability Index

Another method used to evaluate projects is called the **profitability index**. It is the ratio of the present value of the future expected cash flows *after* initial investment divided by the amount of the initial investment. The profitability index can be represented like this:

$$\text{Profitability index (PI)} = \frac{\text{PV of cash flows } \textit{subsequent} \text{ to initial investment}}{\text{Initial investment}}$$

EXAMPLE 6.5

**Profitability Index** Hiram Finnegan Inc. (HFI) applies a 12 percent discount rate to two investment opportunities.

| Project | Cash Flows ($000,000) $C_0$ | $C_1$ | $C_2$ | PV @ 12% of Cash Flows Subsequent to Initial Investment ($000,000) | Profitability Index | NPV @12% ($000,000) |
|---|---|---|---|---|---|---|
| 1 | −$20 | $70 | $10 | $70.5 | 3.53 | $50.5 |
| 2 | − 10 | 15 | 40 | 45.3 | 4.53 | 35.3 |

## *Calculation of Profitability Index*

The profitability index is calculated for project 1 as follows. The present value of the cash flows *after* the initial investment is:

$$\$70.5 = \frac{\$70}{1.12} + \frac{\$10}{(1.12)^2}$$

The profitability index is obtained by dividing this result by the initial investment of $20. This yields:

$$3.53 = \frac{\$70.5}{\$20}$$

**Application of the Profitability Index** How do we use the profitability index? We consider three situations:

1. *Independent projects*: Assume that HFI's two projects are independent. According to the NPV rule, both projects should be accepted because NPV is positive in each case. The profitability index (PI) is greater than 1 whenever the NPV is positive. Thus, the PI *decision rule* is
   - Accept an independent project if PI $>$ 1.
   - Reject it if PI $<$ 1.
2. *Mutually exclusive projects*: Let us now assume that HFI can only accept one of its two projects. NPV analysis says accept project 1 because this project has the bigger NPV. Because project 2 has the higher PI, the profitability index leads to the wrong selection.

The problem with the profitability index for mutually exclusive projects is the same as the scale problem with the IRR that we mentioned earlier. Project 2 is smaller than project 1. Because the PI is a ratio, this index misses the fact that project 1 has a larger investment than project 2 has. Thus, like IRR, PI ignores differences of scale for mutually exclusive projects.

However, like IRR, the flaw with the PI approach can be corrected using incremental analysis. We write the incremental cash flows after subtracting project 2 from project 1 as follows:

| Project | Cash Flows ($000,000) $C_0$ | $C_1$ | $C_2$ | PV @ 12% of Cash Flows Subsequent to Initial Investment ($000,000) | Profitability Index | NPV @12% ($000,000) |
|---|---|---|---|---|---|---|
| 1–2 | −$10 | $55 | −$30 | $25.2 | 2.52 | $15.2 |

Because the profitability index on the incremental cash flows is greater than 1.0, we should choose the bigger project—that is, project 1. This is the same decision we get with the NPV approach.

3. *Capital rationing*: The first two cases implicitly assumed that HFI could always attract enough capital to make any profitable investments. Now consider the case when the firm does not have enough capital to fund all positive NPV projects. This is the case of **capital rationing**.

Imagine that the firm has a third project, as well as the first two. Project 3 has the following cash flows:

| Project | Cash Flows ($000,000) $C_0$ | $C_1$ | $C_2$ | PV @ 12% of Cash Flows Subsequent to Initial Investment ($000,000) | Profitability Index | NPV @12% ($000,000) |
|---|---|---|---|---|---|---|
| 3 | −$10 | −$5 | $60 | $43.4 | 4.34 | $33.4 |

Further, imagine that (1) the projects of Hiram Finnegan Inc. are independent, but (2) the firm has only $20 million to invest. Because project 1 has an initial investment of $20 million, the firm cannot select both this project and another one. Conversely, because projects 2 and 3 have initial investments of $10 million each, both these projects can be chosen. In other words, the cash constraint forces the firm to choose either project 1 or projects 2 and 3.

What should the firm do? Individually, projects 2 and 3 have lower NPVs than project 1 has. However, when the NPVs of projects 2 and 3 are added together, the sum is higher than the NPV of project 1. Thus, common sense dictates that projects 2 and 3 should be accepted.

What does our conclusion have to say about the NPV rule or the PI rule? In the case of limited funds, we cannot rank projects according to their NPVs. Instead we should rank them according to the ratio of present value to initial investment. This is the PI rule. Both project 2 and project 3 have higher PI ratios than does project 1. Thus they should be ranked ahead of project 1 when capital is rationed.

The usefulness of the profitability index under capital rationing can be explained in military terms. The Pentagon speaks highly of a weapon with a lot of "bang for the buck." In capital budgeting, the profitability index measures the bang (the dollar return) for the buck invested. Hence it is useful for capital rationing.

It should be noted that the profitability index does not work if funds are also limited beyond the initial time period. For example, if heavy cash outflows elsewhere in the firm were to occur at date 1, project 3, which also has a cash outflow at date 1, might need to be rejected. In other words, the profitability index cannot handle capital rationing over multiple time periods.

In addition, what economists term *indivisibilities* may reduce the effectiveness of the PI rule. Imagine that HFI has $30 million available for capital investment, not just $20 million. The firm now has enough cash for projects 1 and 2. Because the sum of the NPVs of these two projects is greater than the sum of the NPVs of projects 2 and 3, the firm would be better served by accepting projects 1 and 2. But because projects 2 and 3 still have the highest profitability indexes, the PI rule now leads to the wrong decision. Why does the PI rule lead us astray here? The key is that projects 1 and 2 use up all of the $30 million, whereas projects 2 and 3 have a combined initial investment of only $20 million (= $10 + 10). If projects 2 and 3 are accepted, the remaining $10 million must be left in the bank.

This situation points out that care should be exercised when using the profitability index in the real world. Nevertheless, while not perfect, the profitability index goes a long way toward handling capital rationing.

## 6.8 The Practice of Capital Budgeting

So far this chapter has asked "Which capital budgeting methods should companies be using?" An equally important question is this: Which methods *are* companies using? Table 6.4 helps answer this question. As can be seen from the table, approximately three-quarters of U.S. and Canadian companies use the IRR and NPV methods. This is not surprising, given the theoretical advantages of these approaches. Over half of these companies use the payback method, a rather surprising result given the conceptual problems with this approach. And while discounted payback represents a theoretical improvement over regular payback,

**Table 6.4**
**Percentage of CFOs Who Always or Almost Always Use a Given Technique**

| | % Always or Almost Always |
|---|---|
| Internal rate of return (IRR) | 75.6% |
| Net present value (NPV) | 74.9 |
| Payback method | 56.7 |
| Discounted payback | 29.5 |
| Accounting rate of return | 30.3 |
| Profitability index | 11.9 |

SOURCE: Figure 2 from John R. Graham and Campbell R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60 (2001). Based on a survey of 392 CFOs.

## In Their Own Words

### *KITCHEN CONFIDENTIAL: ADVENTURES IN THE CULINARY UNDERBELLY* BY ANTHONY BOURDAIN (BLOOMSBURY PRESS, 2000)

To want to own a restaurant can be a strange and terrible affliction. What causes such a destructive urge in so many otherwise sensible people? Why would anyone who has worked hard, saved money, and often been successful in other fields want to pump their hard-earned cash down a hole that statistically, at least, will almost surely prove dry? Why venture into an industry with enormous fixed expenses (rent, electricity, gas, water, linen, maintenance, insurance, license fees, trash removal, etc.), a notoriously transient and unstable workforce, and a highly perishable inventory of assets? The chances of ever seeing a return on your investment are about one in five. What insidious spongiform bacteria so riddles the brains of men and women that they stand there on the tracks, watching the lights of the oncoming locomotive, knowing full well it will eventually run them over? After all these years in the business, I still don't know.

Anthony Bourdain is also the author of the novels *Bone in the Throat*, *Gone Bamboo*, and The Bobby Gold Stories. He is the executive chef at Brasserie Les Halles in New York.

the usage here is far less. Perhaps companies are attracted to the user-friendly nature of payback. In addition, the flaws of this approach, as mentioned in the current chapter, may be relatively easy to correct. For example, while the payback method ignores all cash flows after the payback period, an alert manager can make ad hoc adjustments for a project with back-loaded cash flows.

Capital expenditures by individual corporations can add up to enormous sums for the economy as a whole. For example, in late 2005, Royal Dutch Shell announced it expected to increase its capital spending in 2006 to $19 billion, an increase of 17 percent over the previous year. About the same time, competitor Chevron Corp. announced it would increase its capital budget for 2006 to $14.8 billion, up from $11 billion in 2005. Other companies with large capital spending budgets in 2006 were ConocoPhillips, which projected capital spending of $11.4 billion, and Canadian-based Suncor Energy, which projected capital spending of $3.5 billion.

Capital spending is often an industrywide occurrence. For example, in 2006, capital spending by dynamic random access memory (DRAM) chip makers was expected to reach $16.84 billion. This amount represented only a 5 percent increase from 2005 and was a major slowdown for capital spending growth. From 2003 to 2004, the DRAM industry's capital spending had grown by an astonishing 65 percent.

According to information released by the Census Bureau in 2006, capital investment for the economy as a whole was actually $1.05 trillion in 2004, $975 billion in 2003, and $953 billion in 2002. The totals for the three years therefore were about $3 trillion! Given the sums at stake, it is not too surprising that careful analysis of capital expenditures is something at which successful corporations seek to become adept.

One might expect the capital budgeting methods of large firms to be more sophisticated than the methods of small firms. After all, large firms have the financial resources to hire more sophisticated employees. Table 6.5 provides some support for this idea. Here firms indicate frequency of use of the various capital budgeting methods on a scale of 0 (never) to 4 (always). Both the IRR and NPV methods are used more frequently, and payback less frequently, in large firms than in small firms. Conversely, large and small firms employ the last three approaches about equally.

**Table 6.5**

**Frequency of Use of Various Capital Budgeting Methods**

| | Large Firms | Small Firms |
|---|---|---|
| Internal rate of return (IRR) | 3.41 | 2.87 |
| Net present value (NPV) | 3.42 | 2.83 |
| Payback method | 2.25 | 2.72 |
| Discounted payback | 1.55 | 1.58 |
| Accounting rate of return | 1.25 | 1.41 |
| Profitability index | 0.75 | 0.78 |

Firms indicate frequency of use on a scale from 0 (never) to 4 (always). Numbers in table are averages across respondents.
SOURCE: Table 2 from Graham and Harvey (2001), op. cit.

The use of quantitative techniques in capital budgeting varies with the industry. As one would imagine, firms that are better able to estimate cash flows are more likely to use NPV. For example, estimation of cash flow in certain aspects of the oil business is quite feasible. Because of this, energy-related firms were among the first to use NPV analysis. Conversely, the cash flows in the motion picture business are very hard to project. The grosses of the great hits like *Titanic*, *Harry Potter*, and *Star Wars* were far, far greater than anyone imagined. The big failures like *Alamo* and *Waterworld* were unexpected as well. Because of this, NPV analysis is frowned upon in the movie business.

How does Hollywood perform capital budgeting? The information that a studio uses to accept or reject a movie idea comes from the *pitch*. An independent movie producer schedules an extremely brief meeting with a studio to pitch his or her idea for a movie. Consider the following four paragraphs of quotes concerning the pitch from the thoroughly delightful book *Reel Power*:[12]

> "They [studio executives] don't want to know too much," says Ron Simpson. "They want to know concept. . . . They want to know what the three-liner is, because they want it to suggest the ad campaign. They want a title. . . . They don't want to hear any esoterica. And if the meeting lasts more than five minutes, they're probably not going to do the project."
>
> "A guy comes in and says this is my idea: '*Jaws* on a spaceship,' " says writer Clay Frohman (*Under Fire*). "And they say, 'Brilliant, fantastic.' Becomes *Alien*. That is *Jaws* on a spaceship, ultimately. . . . And that's it. That's all they want to hear. Their attitude is 'Don't confuse us with the details of the story.' "
>
> ". . . Some high-concept stories are more appealing to the studios than others. The ideas liked best are sufficiently original that the audience will not feel it has already seen the movie, yet similar enough to past hits to reassure executives wary of anything too far-out. Thus, the frequently used shorthand: It's *Flashdance* in the country (*Footloose*) or *High Noon* in outer space (*Outland*)."
>
> ". . . One gambit not to use during a pitch," says executive Barbara Boyle, "is to talk about big box-office grosses your story is sure to make. Executives know as well as anyone that it's impossible to predict how much money a movie will make, and declarations to the contrary are considered pure malarkey."

[12]Mark Litwak, *Reel Power: The Struggle for Influence and Success in the New Hollywood* (New York: William Morrow and Company, Inc., 1986), pp. 73, 74, and 77.

## Summary and Conclusions

1. In this chapter, we covered different investment decision rules. We evaluated the most popular alternatives to the NPV: the payback period, the discounted payback period, the accounting rate of return, the internal rate of return, and the profitability index. In doing so we learned more about the NPV.
2. While we found that the alternatives have some redeeming qualities, when all is said and done, they are not the NPV rule; for those of us in finance, that makes them decidedly second-rate.
3. Of the competitors to NPV, IRR must be ranked above both payback and accounting rate of return. In fact, IRR always reaches the same decision as NPV in the normal case where the initial outflows of an independent investment project are followed only by a series of inflows.
4. We classified the flaws of IRR into two types. First, we considered the general case applying to both independent and mutually exclusive projects. There appeared to be two problems here:
   - **a.** Some projects have cash inflows followed by one or more outflows. The IRR rule is inverted here: One should accept when the IRR is *below* the discount rate.
   - **b.** Some projects have a number of changes of sign in their cash flows. Here, there are likely to be multiple internal rates of return. The practitioner must use either NPV or modified internal rate of return here.
5. Next, we considered the specific problems with the NPV for mutually exclusive projects. We showed that, due to differences in either size or timing, the project with the highest IRR need not have the highest NPV. Hence, the IRR rule should not be applied. (Of course, NPV can still be applied.)

   However, we then calculated incremental cash flows. For ease of calculation, we suggested subtracting the cash flows of the smaller project from the cash flows of the larger project. In that way the incremental initial cash flow is negative. One can always reach a correct decision by accepting the larger project if the incremental IRR is greater than the discount rate.
6. We described capital rationing as the case where funds are limited to a fixed dollar amount. With capital rationing the profitability index is a useful method of adjusting the NPV.

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## Concept Questions

1. **Payback Period and Net Present Value** If a project with conventional cash flows has a payback period less than the project's life, can you definitively state the algebraic sign of the NPV? Why or why not? If you know that the discounted payback period is less than the project's life, what can you say about the NPV? Explain.
2. **Net Present Value** Suppose a project has conventional cash flows and a positive NPV. What do you know about its payback? Its discounted payback? Its profitability index? Its IRR? Explain.
3. **Comparing Investment Criteria** Define each of the following investment rules and discuss any potential shortcomings of each. In your definition, state the criterion for accepting or rejecting independent projects under each rule.
   - **a.** Payback period.
   - **b.** Average accounting return.
   - **c.** Internal rate of return.
   - **d.** Profitability index.
   - **e.** Net present value.
4. **Payback and Internal Rate of Return** A project has perpetual cash flows of $C$ per period, a cost of $I$, and a required return of $R$. What is the relationship between the project's payback and its IRR? What implications does your answer have for long-lived projects with relatively constant cash flows?

5. **International Investment Projects** In November 2004, automobile manufacturer Honda announced plans to build an automatic transmission plant in Georgia and expand its transmission plant in Ohio. Honda apparently felt that it would be better able to compete and create value with U.S.-based facilities. Other companies such as Fuji Film and Swiss chemical company Lonza have reached similar conclusions and taken similar actions. What are some of the reasons that foreign manufacturers of products as diverse as automobiles, film, and chemicals might arrive at this same conclusion?

6. **Capital Budgeting Problems** What are some of the difficulties that might come up in actual applications of the various criteria we discussed in this chapter? Which one would be the easiest to implement in actual applications? The most difficult?

7. **Capital Budgeting in Not-for-Profit Entities** Are the capital budgeting criteria we discussed applicable to not-for-profit corporations? How should such entities make capital budgeting decisions? What about the U.S. government? Should it evaluate spending proposals using these techniques?

8. **Net Present Value** The investment in project *A* is $1 million, and the investment in project *B* is $2 million. Both projects have a unique internal rate of return of 20 percent. Is the following statement true or false?

   For any discount rate from 0 percent to 20 percent, project *B* has an NPV twice as great as that of project *A*.

   Explain your answer.

9. **Net Present Value versus Profitability Index** Consider the following two mutually exclusive projects available to Global Investments, Inc.:

| | $C_0$ | $C_1$ | $C_2$ | Profitability Index | NPV |
|---|---|---|---|---|---|
| *A* | −$1,000 | $1,000 | $500 | 1.32 | $322 |
| *B* | −500 | 500 | 400 | 1.57 | 285 |

The appropriate discount rate for the projects is 10 percent. Global Investments chose to undertake project *A*. At a luncheon for shareholders, the manager of a pension fund that owns a substantial amount of the firm's stock asks you why the firm chose project *A* instead of project *B* when project *B* has a higher profitability index.

How would you, the CFO, justify your firm's action? Are there any circumstances under which Global Investments should choose project *B*?

10. **Internal Rate of Return** Projects *A* and *B* have the following cash flows:

| Year | Project *A* | Project *B* |
|---|---|---|
| 0 | −$1,000 | −$2,000 |
| 1 | C1A | C1B |
| 2 | C2A | C2B |
| 3 | C3A | C3B |

   **a.** If the cash flows from the projects are identical, which of the two projects would have a higher IRR? Why?

   **b.** If C1B = 2C1A, C2B = 2C2A, and C3B = 2C3A, then is $IRR_A = IRR_B$?

11. **Net Present Value** You are evaluating project *A* and project *B*. Project *A* has a short period of future cash flows, while project *B* has relatively long future cash flows. Which project will be more sensitive to changes in the required return? Why?

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12. **Modified Internal Rate of Return** One of the less flattering interpretations of the acronym MIRR is "meaningless internal rate of return." Why do you think this term is applied to MIRR?

13. **Net Present Value** It is sometimes stated that "the net present value approach assumes reinvestment of the intermediate cash flows at the required return." Is this claim correct? To answer, suppose you calculate the NPV of a project in the usual way. Next, suppose you do the following:
    **a.** Calculate the future value (as of the end of the project) of all the cash flows other than the initial outlay assuming they are reinvested at the required return, producing a single future value figure for the project.
    **b.** Calculate the NPV of the project using the single future value calculated in the previous step and the initial outlay. It is easy to verify that you will get the same NPV as in your original calculation only if you use the required return as the reinvestment rate in the previous step.

14. **Internal Rate of Return** It is sometimes stated that "the internal rate of return approach assumes reinvestment of the intermediate cash flows at the internal rate of return." Is this claim correct? To answer, suppose you calculate the IRR of a project in the usual way. Next, suppose you do the following:
    **a.** Calculate the future value (as of the end of the project) of all the cash flows other than the initial outlay assuming they are reinvested at the IRR, producing a single future value figure for the project.
    **b.** Calculate the IRR of the project using the single future value calculated in the previous step and the initial outlay. It is easy to verify that you will get the same IRR as in your original calculation only if you use the IRR as the reinvestment rate in the previous step.

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## Questions and Problems

**BASIC**
**(Questions 1–10)**

1. **Calculating Payback Period and NPV** Fuji Software, Inc., has the following mutually exclusive projects.

| Year | Project *A* | Project *B* |
|---|---|---|
| 0 | −$7,500 | −$5,000 |
| 1 | 4,000 | 2,500 |
| 2 | 3,500 | 1,200 |
| 3 | 1,500 | 3,000 |

    **a.** Suppose Fuji's payback period cutoff is two years. Which of these two projects should be chosen?
    **b.** Suppose Fuji uses the NPV rule to rank these two projects. Which project should be chosen if the appropriate discount rate is 15 percent?

2. **Calculating Payback** An investment project provides cash inflows of $840 per year for eight years. What is the project payback period if the initial cost is $3,000? What if the initial cost is $5,000? What if it is $7,000?

3. **Calculating Discounted Payback** An investment project has annual cash inflows of $7,000, $7,500, $8,000, and $8,500, and a discount rate of 14 percent. What is the discounted payback period for these cash flows if the initial cost is $8,000? What if the initial cost is $13,000? What if it is $18,000?

4. **Calculating Discounted Payback** An investment project costs $10,000 and has annual cash flows of $2,100 for six years. What is the discounted payback period if the discount rate is 0 percent? What if the discount rate is 5 percent? If it is 15 percent?

**5.** **Average Accounting Return** Your firm is considering purchasing a machine with the following annual, end-of-year, book investment accounts:

| | Purchase Date | Year 1 | Year 2 | Year 3 | Year 4 |
|---|---|---|---|---|---|
| Gross investment | $16,000 | $16,000 | $16,000 | $16,000 | $16,000 |
| Less: Accumulated depreciation | 0 | 4,000 | 8,000 | 12,000 | 16,000 |
| Net investment | $16,000 | $12,000 | $ 8,000 | $ 4,000 | $ 0 |

The machine generates, on average, $4,500 per year in additional net income.

**a.** What is the average accounting return for this machine?

**b.** What three flaws are inherent in this decision rule?

**6.** **Average Accounting Return** The Bluerock Group has invested $8,000 in a high-tech project lasting three years. Depreciation is $4,000, $2,500, and $1,500 in years 1, 2, and 3, respectively. The project generates pretax income of $2,000 each year. The pretax income already includes the depreciation expense. If the tax rate is 25 percent, what is the project's average accounting return (AAR)?

**7.** **Calculating IRR** Teddy Bear Planet, Inc., has a project with the following cash flows:

| Year | Cash Flows ($) |
|---|---|
| 0 | −$8,000 |
| 1 | 4,000 |
| 2 | 3,000 |
| 3 | 2,000 |

The company evaluates all projects by applying the IRR rule. If the appropriate interest rate is 8 percent, should the company accept the project?

**8.** **Calculating IRR** Compute the internal rate of return for the cash flows of the following two projects:

| | Cash Flows ($) | |
|---|---|---|
| **Year** | **Project *A*** | **Project *B*** |
| 0 | −$2,000 | −$1,500 |
| 1 | 1,000 | 500 |
| 2 | 1,500 | 1,000 |
| 3 | 2,000 | 1,500 |

**9.** **Calculating Profitability Index** Bill plans to open a self-serve grooming center in a storefront. The grooming equipment will cost $160,000, to be paid immediately. Bill expects aftertax cash inflows of $40,000 annually for seven years, after which he plans to scrap the equipment and retire to the beaches of Nevis. The first cash inflow occurs at the end of the first year. Assume the required return is 15 percent. What is the project's PI? Should it be accepted?

**10.** **Calculating Profitability Index** Suppose the following two independent investment opportunities are available to Greenplain, Inc. The appropriate discount rate is 10 percent.

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| Year | Project Alpha | Project Beta |
|---|---|---|
| 0 | −$500 | −$2,000 |
| 1 | 300 | 300 |
| 2 | 700 | 1,800 |
| 3 | 600 | 1,700 |

**a.** Compute the profitability index for each of the two projects.
**b.** Which project(s) should Greenplain accept based on the profitability index rule?

**INTERMEDIATE (Questions 11–23)**

**11. Cash Flow Intuition** A project has an initial cost of *I*, has a required return of *R*, and pays *C* annually for *N* years.
**a.** Find *C* in terms of *I* and *N* such that the project has a payback period just equal to its life.
**b.** Find *C* in terms of *I*, *N*, and *R* such that this is a profitable project according to the NPV decision rule.
**c.** Find *C* in terms of *I*, *N*, and *R* such that the project has a benefit–cost ratio of 2.

**12. Problems with IRR** Suppose you are offered $5,000 today but must make the following payments:

| Year | Cash Flows ($) |
|---|---|
| 0 | $5,000 |
| 1 | −2,500 |
| 2 | −2,000 |
| 3 | −1,000 |
| 4 | −1,000 |

**a.** What is the IRR of this offer?
**b.** If the appropriate discount rate is 10 percent, should you accept this offer?
**c.** If the appropriate discount rate is 20 percent, should you accept this offer?
**d.** What is the NPV of the offer if the appropriate discount rate is 10 percent? 20 percent?
**e.** Are the decisions under the NPV rule in part (d) consistent with those of the IRR rule?

**13. NPV versus IRR** Consider the following cash flows on two mutually exclusive projects for the Bahamas Recreation Corporation (BRC). Both projects require an annual return of 15 percent.

| Year | Deepwater Fishing | New Submarine Ride |
|---|---|---|
| 0 | −$600,000 | −$1,800,000 |
| 1 | 270,000 | 1,000,000 |
| 2 | 350,000 | 700,000 |
| 3 | 300,000 | 900,000 |

As a financial analyst for BRC, you are asked the following questions:
**a.** If your decision rule is to accept the project with the greater IRR, which project should you choose?
**b.** Because you are fully aware of the IRR rule's scale problem, you calculate the incremental IRR for the cash flows. Based on your computation, which project should you choose?
**c.** To be prudent, you compute the NPV for both projects. Which project should you choose? Is it consistent with the incremental IRR rule?

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14. **Problems with Profitability Index** The Robb Computer Corporation is trying to choose between the following two mutually exclusive design projects:

| Year | Cash Flow (I) | Cash Flow (II) |
|---|---|---|
| 0 | −$30,000 | −$5,000 |
| 1 | 15,000 | 2,800 |
| 2 | 15,000 | 2,800 |
| 3 | 15,000 | 2,800 |

**a.** If the required return is 10 percent and Robb Computer applies the profitability index decision rule, which project should the firm accept?
**b.** If the company applies the NPV decision rule, which project should it take?
**c.** Explain why your answers in (a) and (b) are different.

15. **Problems with IRR** Cutler Petroleum, Inc., is trying to evaluate a generation project with the following cash flows:

| Year | Cash Flow |
|---|---|
| 0 | −$28,000,000 |
| 1 | 53,000,000 |
| 2 | −8,000,000 |

**a.** If the company requires a 10 percent return on its investments, should it accept this project? Why?
**b.** Compute the IRR for this project. How many IRRs are there? If you apply the IRR decision rule, should you accept the project or not? What's going on here?

16. **Comparing Investment Criteria** Mario Brothers, a game manufacturer, has a new idea for an adventure game. It can market the game either as a traditional board game or as an interactive CD-ROM, but not both. Consider the following cash flows of the two mutually exclusive projects for Mario Brothers. Assume the discount rate for Mario Brothers is 10 percent.

| Year | Board Game | CD-ROM |
|---|---|---|
| 0 | −$300 | −$1,500 |
| 1 | 400 | 1,100 |
| 2 | 100 | 800 |
| 3 | 100 | 400 |

**a.** Based on the payback period rule, which project should be chosen?
**b.** Based on the NPV, which project should be chosen?
**c.** Based on the IRR, which project should be chosen?
**d.** Based on the incremental IRR, which project should be chosen?

17. **Profitability Index versus NPV** Hanmi Group, a consumer electronics conglomerate, is reviewing its annual budget in wireless technology. It is considering investments in three different technologies to develop wireless communication devices. Consider the following cash

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flows of the three independent projects for Hanmi. Assume the discount rate for Hanmi is 10 percent. Further, Hanmi Group has only $30 million to invest in new projects this year.

| | Cash Flows (in $ millions) | | |
|---|---|---|---|
| **Year** | **CDMA** | **G4** | **Wi-Fi** |
| 0 | −$10 | −$20 | −$ 30 |
| 1 | 25 | 20 | 20 |
| 2 | 15 | 50 | 40 |
| 3 | 5 | 40 | 100 |

**a.** Based on the profitability index decision rule, rank these investments.
**b.** Based on the NPV, rank these investments.
**c.** Based on your findings in (a) and (b), what would you recommend to the CEO of Hanmi Group and why?

**18.** **Comparing Investment Criteria** Consider the following cash flows of two mutually exclusive projects for AZ-Motorcars. Assume the discount rate for AZ-Motorcars is 10 percent.

| Year | AZM Mini-SUV | AZF Full-SUV |
|---|---|---|
| 0 | −$200,000 | −$500,000 |
| 1 | 200,000 | 200,000 |
| 2 | 150,000 | 300,000 |
| 3 | 150,000 | 300,000 |

**a.** Based on the payback period, which project should be taken?
**b.** Based on the NPV, which project should be taken?
**c.** Based on the IRR, which project should be taken?
**d.** Based on this analysis, is incremental IRR analysis necessary? If yes, please conduct the analysis.

**19.** **Comparing Investment Criteria** The treasurer of Amaro Canned Fruits, Inc., has projected the cash flows of projects *A*, *B*, and *C* as follows.

| Year | Project *A* | Project *B* | Project *C* |
|---|---|---|---|
| 0 | −$100,000 | −$200,000 | −$100,000 |
| 1 | 70,000 | 130,000 | 75,000 |
| 2 | 70,000 | 130,000 | 60,000 |

Suppose the relevant discount rate is 12 percent a year.
**a.** Compute the profitability index for each of the three projects.
**b.** Compute the NPV for each of the three projects.
**c.** Suppose these three projects are independent. Which project(s) should Amaro accept based on the profitability index rule?
**d.** Suppose these three projects are mutually exclusive. Which project(s) should Amaro accept based on the profitability index rule?
**e.** Suppose Amaro's budget for these projects is $300,000. The projects are not divisible. Which project(s) should Amaro accept?

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**20. Comparing Investment Criteria** Consider the following cash flows of two mutually exclusive projects for Tokyo Rubber Company. Assume the discount rate for Tokyo Rubber Company is 10 percent.

| Year | Dry Prepreg | Solvent Prepreg |
|---|---|---|
| 0 | −$1,000,000 | −$500,000 |
| 1 | 600,000 | 300,000 |
| 2 | 400,000 | 500,000 |
| 3 | 1,000,000 | 100,000 |

**a.** Based on the payback period, which project should be taken?
**b.** Based on the NPV, which project should be taken?
**c.** Based on the IRR, which project should be taken?
**d.** Based on this analysis, is incremental IRR analysis necessary? If yes, please conduct the analysis.

**21. Comparing Investment Criteria** Consider two mutually exclusive new product launch projects that Nagano Golf is considering. Assume the discount rate for Nagano Golf is 15 percent.

Project *A*: Nagano NP-30.
Professional clubs that will take an initial investment of $100,000 at time 0.
Next five years (years 1–5) of sales will generate a consistent cash flow of $40,000 per year.
Introduction of new product at year 6 will terminate further cash flows from this project.

Project *B*: Nagano NX-20.
High-end amateur clubs that will take an initial investment of $30,000 at time 0.
Cash flow at year 1 is $20,000. In each subsequent year cash flow will grow at 15 percent per year.
Introduction of new product at year 6 will terminate further cash flows from this project.

| Year | NP-30 | NX-20 |
|---|---|---|
| 0 | −$100,000 | −$30,000 |
| 1 | 40,000 | 20,000 |
| 2 | 40,000 | 23,000 |
| 3 | 40,000 | 26,450 |
| 4 | 40,000 | 30,418 |
| 5 | 40,000 | 34,980 |

Please fill in the following table:

| | NP-30 | NX-20 | Implications |
|---|---|---|---|
| NPV | | | |
| IRR | | | |
| Incremental IRR | | | |
| PI | | | |

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22. **Comparing Investment Criteria** Consider two mutually exclusive R&D projects that ADM is considering. Assume the discount rate for ADM is 15 percent.

Project *A*: Server CPU .13 micron processing project.
By shrinking the die size to .13 micron, ADM will be able to offer server CPU chips with lower power consumption and heat generation, meaning faster CPUs.

Project *B*: New telecom chip project.
Entry into this industry will require introduction of a new chip for cellphones. The know-how will require a lot of upfront capital, but success of the project will lead to large cash flows later on.

| Year | *A* | *B* |
|---|---|---|
| 0 | −$100,000 | −$200,000 |
| 1 | 50,000 | 60,000 |
| 2 | 50,000 | 60,000 |
| 3 | 40,000 | 60,000 |
| 4 | 30,000 | 100,000 |
| 5 | 20,000 | 200,000 |

Please fill in the following table:

| | *A* | *B* | Implications |
|---|---|---|---|
| NPV | | | |
| IRR | | | |
| Incremental IRR | | | |
| PI | | | |

23. **Comparing Investment Criteria** You are a senior manager at Poeing Aircraft and have been authorized to spend up to $200,000 for projects. The three projects you are considering have the following characteristics:

Project *A*: Initial investment of $150,000. Cash flow of $50,000 at year 1 and $100,000 at year 2. This is a plant expansion project, where the required rate of return is 10 percent.

Project *B*: Initial investment of $200,000. Cash flow of $200,000 at year 1 and $111,000 at year 2. This is a new product development project, where the required rate of return is 20 percent.

Project *C*: Initial investment of $100,000. Cash flow of $100,000 at year 1 and $100,000 at year 2. This is a market expansion project, where the required rate of return is 20 percent.

Assume the corporate discount rate is 10 percent.
Please offer your recommendations, backed by your analysis:

| | *A* | *B* | *C* | Implications |
|---|---|---|---|---|
| Payback | | | | |
| IRR | | | | |
| Incremental IRR | | | | |
| PI | | | | |
| NPV | | | | |

**CHALLENGE (Questions 24–30)**

**24. Payback and NPV** An investment under consideration has a payback of seven years and a cost of $483,000. If the required return is 12 percent, what is the worst-case NPV? The best-case NPV? Explain. Assume the cash flows are conventional.

**25. Multiple IRRs** This problem is useful for testing the ability of financial calculators and computer software. Consider the following cash flows. How many different IRRs are there? (*Hint*: Search between 20 percent and 70 percent.) When should we take this project?

| Year | Cash Flow |
|---|---|
| 0 | −$ 504 |
| 1 | 2,862 |
| 2 | −6,070 |
| 3 | 5,700 |
| 4 | −2,000 |

**26. NPV Valuation** The Yurdone Corporation wants to set up a private cemetery business. According to the CFO, Barry M. Deep, business is "looking up." As a result, the cemetery project will provide a net cash inflow of $50,000 for the firm during the first year, and the cash flows are projected to grow at a rate of 6 percent per year forever. The project requires an initial investment of $780,000.

**a.** If Yurdone requires a 13 percent return on such undertakings, should the cemetery business be started?

**b.** The company is somewhat unsure about the assumption of a 6 percent growth rate in its cash flows. At what constant growth rate would the company just break even if it still required a 13 percent return on investment?

**27. Calculating IRR** The Utah Mining Corporation is set to open a gold mine near Provo, Utah. According to the treasurer, Monty Goldstein, "This is a golden opportunity." The mine will cost $600,000 to open and will have an economic life of 11 years. It will generate a cash inflow of $100,000 at the end of the first year, and the cash inflows are projected to grow at 8 percent per year for the next 10 years. After 11 years, the mine will be abandoned. Abandonment costs will be $50,000 at the end of year 11.

**a.** What is the IRR for the gold mine?

**b.** The Utah Mining Corporation requires a 10 percent return on such undertakings. Should the mine be opened?

**28. Calculating IRR** Consider two streams of cash flows, *A* and *B*. Stream *A*'s first cash flow is $5,000 and is received three years from today. Future cash flows in stream *A* grow by 4 percent in perpetuity. Stream *B*'s first cash flow is −$6,000, is received two years from today, and will continue in perpetuity. Assume that the appropriate discount rate is 12 percent.

**a.** What is the present value of each stream?

**b.** Suppose that the two streams are combined into one project, called *C*. What is the IRR of project *C*?

**c.** What is the correct IRR rule for project *C*?

**29. Calculating Incremental Cash Flows** Darin Clay, the CFO of MakeMoney.com, has to decide between the following two projects:

| Year | Project Million | Project Billion |
|---|---|---|
| 0 | −$1,500 | $-\$I_o$ |
| 1 | $I_o + 200$ | $I_o + 500$ |
| 2 | 1,200 | 1,500 |
| 3 | 1,500 | 2,000 |

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The expected rate of return for either of the two projects is 12 percent. What is the range of initial investment ($I_0$) for which Project Billion is more financially attractive than Project Million?

**30. Problems with IRR** McKeekin Corp. has a project with the following cash flows:

| Year | Cash Flow |
|---|---|
| 0 | $20,000 |
| 1 | −26,000 |
| 2 | 13,000 |

What is the IRR of the project? What is happening here?

Mini Case

## Bullock Gold Mining

Seth Bullock, the owner of Bullock Gold Mining, is evaluating a new gold mine in South Dakota. Dan Dority, the company's geologist, has just finished his analysis of the mine site. He has estimated that the mine would be productive for eight years, after which the gold would be completely mined. Dan has taken an estimate of the gold deposits to Alma Garrett, the company's financial officer. Alma has been asked by Seth to perform an analysis of the new mine and present her recommendation on whether the company should open the new mine.

Alma has used the estimates provided by Dan to determine the revenues that could be expected from the mine. She has also projected the expense of opening the mine and the annual operating expenses. If the company opens the mine, it will cost $500 million today, and it will have a cash outflow of $80 million nine years from today in costs associated with closing the mine and reclaiming the area surrounding it. The expected cash flows each year from the mine are shown in the following table. Bullock Mining has a 12 percent required return on all of its gold mines.

| Year | Cash Flow |
|---|---|
| 0 | −$500,000,000 |
| 1 | 60,000,000 |
| 2 | 90,000,000 |
| 3 | 170,000,000 |
| 4 | 230,000,000 |
| 5 | 205,000,000 |
| 6 | 140,000,000 |
| 7 | 110,000,000 |
| 8 | 70,000,000 |
| 9 | −80,000,000 |

1. Construct a spreadsheet to calculate the payback period, internal rate of return, modified internal rate of return, and net present value of the proposed mine.
2. Based on your analysis, should the company open the mine?
3. Bonus question: Most spreadsheets do not have a built-in formula to calculate the payback period. Write a VBA script that calculates the payback period for a project.